

# Updates: Overlapping and Lensed GWs

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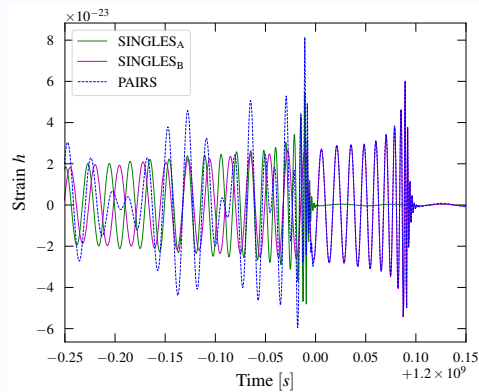
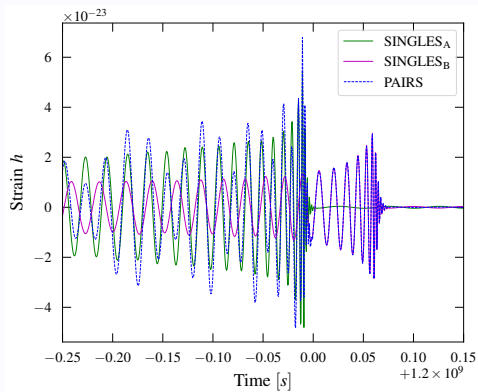
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# Agenda

- 1 Overview
- 2 Introduction
- 3 Analysis Setup
- 4 Results
- 5 Conclusions

# Overlapping Transients



Overlapping Gravitational Wave Signals.

# Study Overview

- Systematically vary key parameters influencing waveform evolution: Chirp mass ratio:  $\mathcal{M}_B/\mathcal{M}_A$ , SNR ratio:  $\text{SNR}_B/\text{SNR}_A$ , Coalescence time difference:  $\Delta t_c$
- Inferences for **Unlensed** (single waveform), **Type II Lensed** (Strong Lensing,  $n_j = 0.5$ ) and **Microlensed** (Isolated point-mass lens).

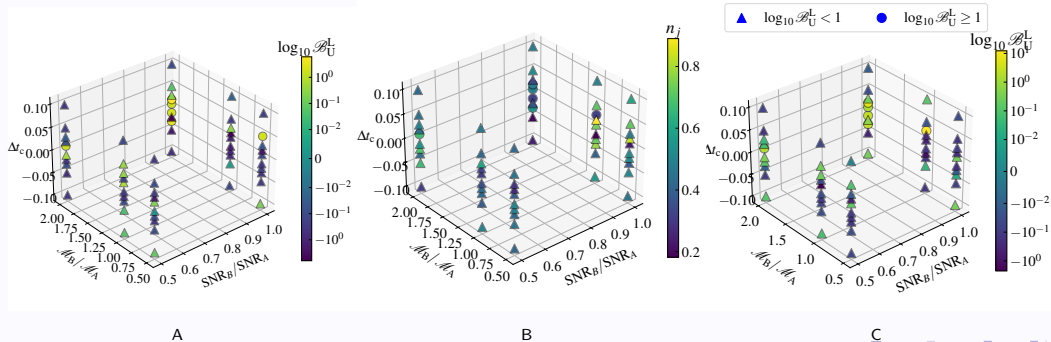
| Individual Case                                                                                                                                                                                                                              | Population Study                                                                                                                                                                                                                        |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <ul style="list-style-type: none"> <li>• <math>\mathcal{M}_B/\mathcal{M}_A \in \{0.5, 1, 2\}</math></li> <li>• <math>\text{SNR}_B/\text{SNR}_A \in \{0.5, 1\}</math></li> <li>• <math>\Delta t_c \in [-0.1, 0.1] \text{ s}</math></li> </ul> | <ul style="list-style-type: none"> <li>• <math>\mathcal{M}_B/\mathcal{M}_A \in [0.1, 10]</math></li> <li>• <math>\text{SNR}_B/\text{SNR}_A \in [0.1, 10]</math></li> <li>• <math>\Delta t_c \in [-0.1, 0.1] \text{ s}</math></li> </ul> |
| Parameter Estimation (for 60 signals).                                                                                                                                                                                                       | Fitting factor ( $\sim \mathcal{O}(5000)$ signals).                                                                                                                                                                                     |

# Type II Lensed Parameter Estimation Inferences

$\mathcal{M}_B/\mathcal{M}_A \in \{0.5, 1, 2\}$ ,  $\text{SNR}_B/\text{SNR}_A \in \{0.5, 1\}$ ,  $\Delta t_c \in [-0.1, 0.1]s$

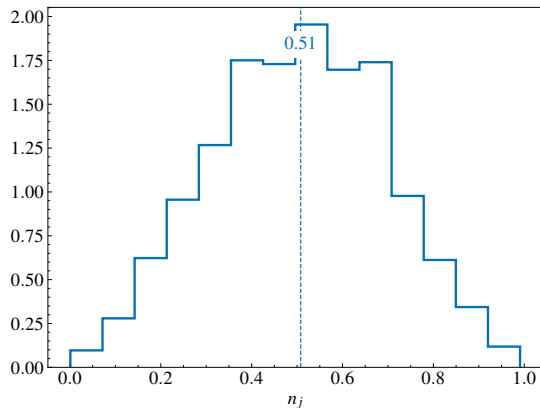
A : Fixed Morse phase shows distinct Bayes factor differences over the unlensed case.

B,C : Allowing the Morse phase to vary improves lensing characterization.



# Population Fitting Factor Results: Type II Lensed Template

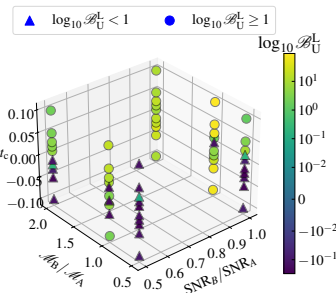
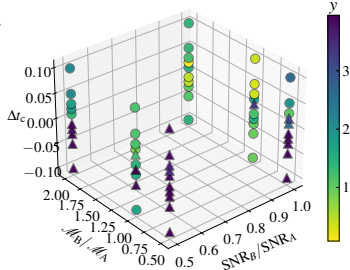
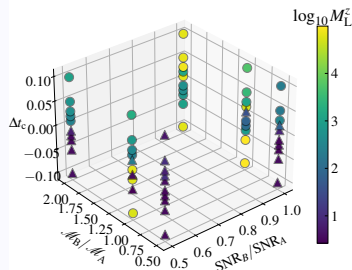
- $\log_{10} \mathcal{B}_U^L > 1$  in a small region of the overlapping parameter space with  $\mathcal{M}_B/\mathcal{M}_A \gtrsim 1$  and  $|\Delta t_c| \leq 0.03$  s.
- Inferred Morse index clustering near  $n_j \simeq 0.5$ , indicative of Type-II lensing, for the cumulative study.



# Microlensed Parameter Estimation Inferences

$\mathcal{M}_B/\mathcal{M}_A \in \{0.5, 1, 2\}$ ,  $\text{SNR}_B/\text{SNR}_A \in \{0.5, 1\}$ ,  $\Delta t_c \in [-0.1, 0.1]s$

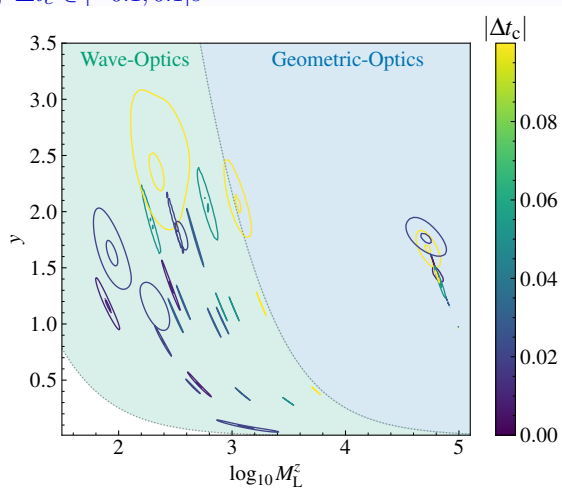
- Microlensed templates yield stronger support for strongly overlapping signals.
- Maximally favored ( $\log_{10} \mathcal{B}_U^L \gg 1$ ) for  $\mathcal{M}_B/\mathcal{M}_A \gtrsim 1$  and equal SNRs, increasing with  $|\Delta t_c|$ .



# Gravitational Microlensing Parameter Estimation Posteriors

$\mathcal{M}_B/\mathcal{M}_A \in \{0.5, 1, 2\}$ ,  $\text{SNR}_B/\text{SNR}_A \in \{0.5, 1\}$ ,  $\Delta t_c \in [-0.1, 0.1]s$

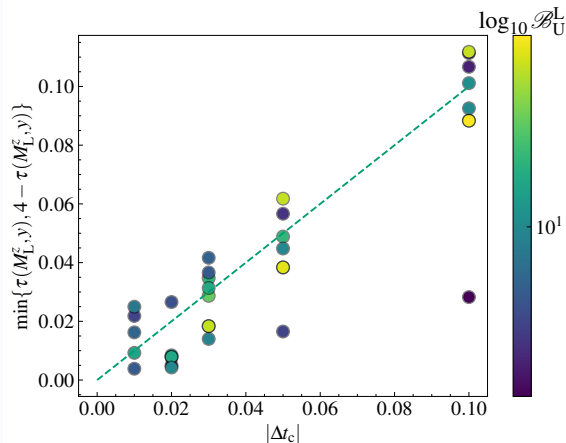
- Inferred lens parameters ( $M_L^z$ ,  $y$ ) show dependencies with  $|\Delta t_c|$ .
- The inferred redshifted lens masses lie in the range  $M_L^z \sim 10^2 - 10^5 M_\odot$  with impact parameters  $y \sim 0.1 - 3$ .



# Microlensing Time Delay

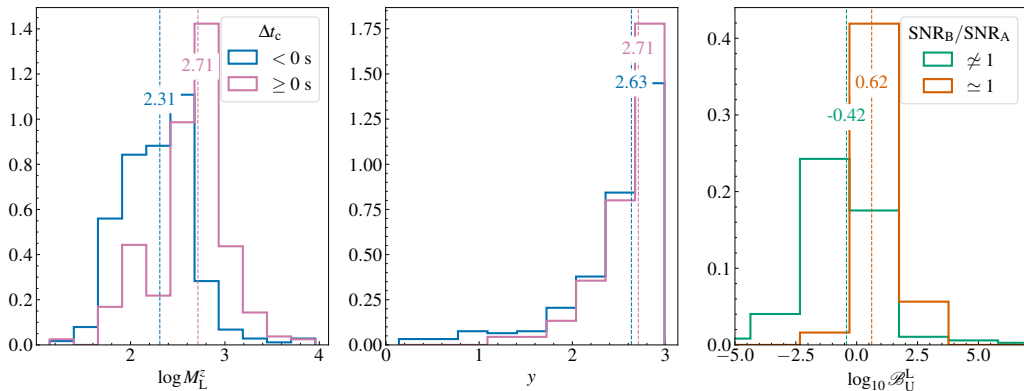
$$\mathcal{M}_B/\mathcal{M}_A \in \{0.5, 1, 2\}, \text{ SNR}_B/\text{SNR}_A \in \{0.5, 1\}, \Delta t_c \in [-0.1, 0.1]s$$

- Inferred time delay of two superimposed microimages is close to the injected  $|\Delta t_c|$  or  $(\delta - |\Delta t_c|)$ .
- False evidence for microlensing signatures, the model produces two superimposed images whose time delay can closely match  $|\Delta t_c|$ .



# Population Fitting Factor Results: Microlensed Template

- FF optimization shows moderate microlensing support when  $\text{SNR}_B/\text{SNR}_A \sim 1$ , consistent with PE trends.



# Conclusions

- The inferred Bayes factor differences depends on relative chirp-mass ratios, relative loudness, difference in coalescence times, and also the absolute SNRs of the overlapping signals.
- Overlapping black-hole binaries with nearly equal chirp masses and comparable loudness are likely to be falsely identified as lensed, and can lead to significant biases in single signal unlensed parameter recovery.
- Advanced parameter estimation methods are essential to disentangle these effects. While our study focuses on ground-based detectors using appropriate detectability thresholds, the findings naturally extend to next-generation GW observatories.

# References

-  Samajdar, A. et al., 2021. *Biases in parameter estimation from overlapping gravitational-wave signals in the third-generation detector era*. Physical Review D, 104(4), p.044003.
-  Relton, P. and Raymond, V., 2021. *Parameter estimation bias from overlapping binary black hole events in second generation interferometers*. Physical Review D, 104(8), p.084039.
-  Janquart, J. et al., 2023. *Analyses of overlapping gravitational wave signals using hierarchical subtraction and joint parameter estimation*. Monthly Notices of the Royal Astronomical Society, 523(2), pp.1699-1710.
-  Veitch, J. and Vecchio, A., 2010. *Bayesian coherent analysis of in-spiral gravitational wave signals with a detector network*. Physical Review D—Particles, Fields, Gravitation, and Cosmology, 81(6), p.062003.
-  Owen, B.J., 1996. *Search templates for gravitational waves from inspiraling binaries: Choice of template spacing*. Physical Review D, 53(12), p.6749.

The background of the slide features a stylized representation of a gravitational well or a black hole. A bright, glowing yellow and orange ring, representing the event horizon, is centered in the lower half of the image. Above this ring, a blue, ethereal, smoke-like or plasma-like structure rises and spirals upwards, creating a sense of dynamic movement. The overall color palette is a mix of soft blues, purples, and warm yellows/oranges, giving it a cosmic and artistic feel.

Thank You!

Questions? Comments?

# Analysis Techniques

- **Parameter Estimation:** Bayesian inference [Veitch, J. and Vecchio, A., 2010]:

$$\mathcal{L}(d|\theta) \propto \exp \left[ -\frac{1}{2} \langle d - h(\theta) | d - h(\theta) \rangle \right]$$

$$\underbrace{\log_{10} \mathcal{B}_U^L}_{\text{Bayes Factor}} = \log_{10} \mathcal{Z}_L - \log_{10} \mathcal{Z}_U, \quad \mathcal{Z}_M = \int d\theta \mathcal{L}(d|\theta, \mathcal{H}_M) \pi_M(\theta|\mathcal{H}_M)$$

- **Fitting Factor:** Maximizing waveform overlap [Owen, B.J., 1996]:

$$\mathcal{M}[h_1, h_2] = \max_{t_c, \Phi_c} \frac{\langle h_1 | h_2 \rangle}{\sqrt{\langle h_1 | h_1 \rangle \langle h_2 | h_2 \rangle}}.$$

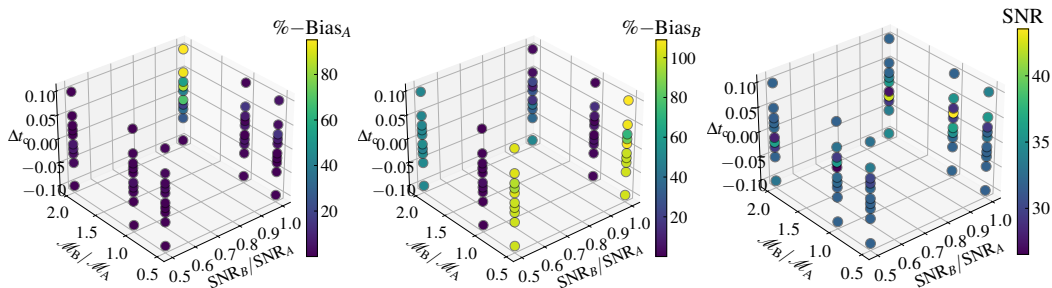
$$\mathcal{F} = \max_{\lambda} \mathcal{M}[h_1, h_2(\lambda)], \quad \log_{10} \mathcal{B}_U^L = (\mathcal{F}_L^2 - \mathcal{F}_U^2) \frac{\text{SNR}^2}{2}$$

# Parameter Estimation Priors

| Parameter                      | PE Priors                                                  |
|--------------------------------|------------------------------------------------------------|
| $n_j$ : Morse Phase            | Fixed: $\delta(n_j - 0.5)$<br>Varying: $\mathcal{U}(0, 1)$ |
| $y$ : Impact Parameter         | POWERLAW $_{\alpha=2}(0.01, 5)$                            |
| $M_L^z$ : Redshifted Lens Mass | Log-Uniform in $[0.1, 10^5] M_\odot$                       |

# Unlensed Parameter Estimation Results

$\mathcal{M}_B/\mathcal{M}_A \in \{0.5, 1, 2\}$ ,  $\text{SNR}_B/\text{SNR}_A \in \{0.5, 1\}$ ,  $\Delta t_c \in [-0.1, 0.1]s$



# Fitting Factor Results: Unlensed Template

