Degeneracies between Overlapping and Lensing GWs

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Nishkal Rao[1], Apratim Ganguly[2], Anupreeta More[2,3], Anuj Mishra[4,2]

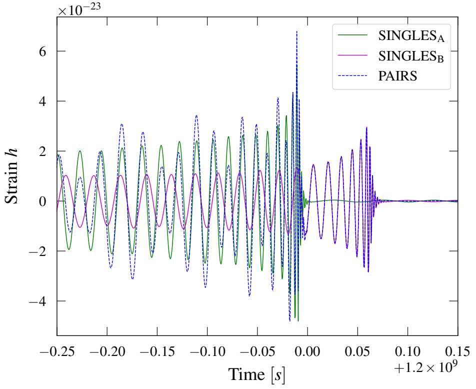
[1]Indian Institute of Science Education and Research (IISER), Pune, India
[2]Inter-University Centre for Astronomy and Astrophysics (IUCAA), Pune, India
[3]Kavli IPMU (WPI), The University of Tokyo, Japan
[4]International Centre for Theoretical Sciences, TIFR, Bangalore, India

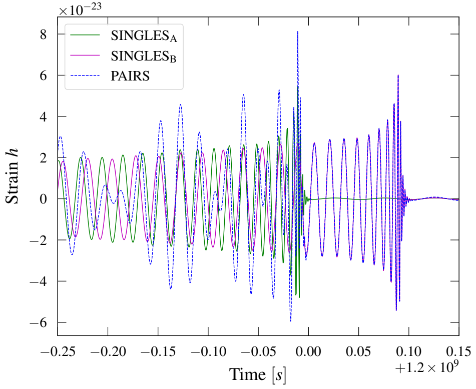
nishkal.rao@students.iiserpune.ac.in

Agenda

1. Overview
2. Introduction
3. Lensing Effects
4. Analysis Setup
5. Results
6. Conclusions

Overlapping Transients





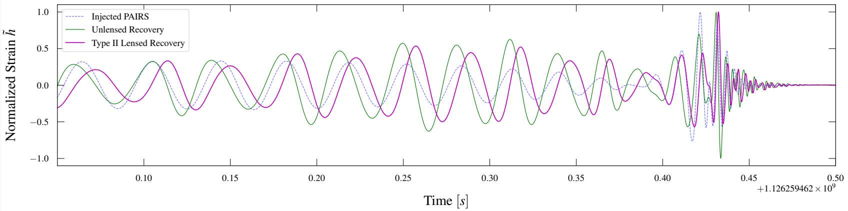
Overlapping Gravitational Wave Signals.

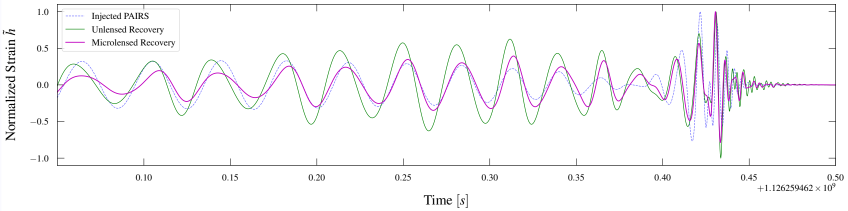
Type II Lensing and Microlensing Effects

Single-Image Lensing Analysis

- **Strong Lensing (Type II):** It is an effect of scaling the signal's amplitude, a temporal delay and an overall phase shift. Resulting in non-trivial properties in the time domain for certain parameters.
- **Microlensing:** Exhibits frequency-dependent amplification due to wave-optics effects. Resemblance to the beating patterns in overlapping signals.

Resemblance with Overlapping transients





Recovery of overlapping transient with Type II Lensing (*Top*) and Microlensing (*Bottom*) effects.

Study Overview

- Systematically vary key parameters influencing waveform evolution:
  - Chirp mass ratio: $\mathcal{M}_B/\mathcal{M}_A$
  - SNR ratio: $\mathrm{SNR_B}/\mathrm{SNR_A}$
  - Coalescence time difference: Δt_c
- Recoveries for **Unlensed** (single waveform), **Type II Lensed** and **Microlensed**.

Individual Case	Population Study
• $\mathcal{M}_B/\mathcal{M}_A \in \{0.5,\ 1,\ 2\}$	• $\mathcal{M}_B/\mathcal{M}_A \in [0.1,\ 10]$
• $\mathrm{SNR_B}/\mathrm{SNR_A} \in \{0.5,\ 1\}$	• $\mathrm{SNR_B}/\mathrm{SNR_A} \in [0.1,\ 10]$
• $\Delta t_c \in [-1,\ 1]\,\mathrm{s}$	• $\Delta t_c \in [-1,\ 1]\,\mathrm{s}$
Parameter estimation (for 60 signals).	Fitting factor ($\sim \mathcal{O}(5000)$ signals).

Analysis Techniques

- **Parameter Estimation:** Bayesian inference [Veitch, J. and Vecchio, A., 2010]:

$$\mathcal{L}(d|\theta) \propto \exp\left[-\frac{1}{2}\langle d - h(\theta)|d - h(\theta)\rangle\right]$$

$$\underbrace{\log_{10}\mathcal{B}_U^L}_{\text{Bayes Factor}} = \log_{10}\mathcal{Z}_L - \log_{10}\mathcal{Z}_U, \quad \mathcal{Z}_M = \int d\theta \mathcal{L}(d|\theta, \mathcal{H}_M)\pi_M(\theta|\mathcal{H}_M)$$

- **Fitting Factor:** Maximizing waveform overlap [Owen, B.J., 1996]:

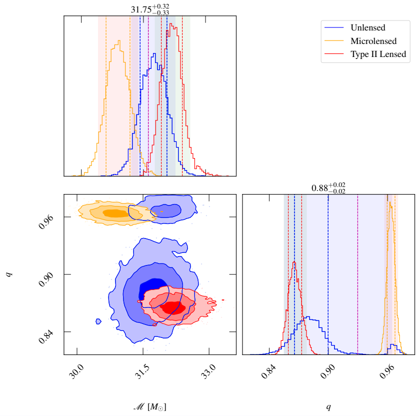
$$\mathcal{M}[h_1, h_2] = \max_{t_c, \Phi_c} \frac{\langle h_1|h_2\rangle}{\sqrt{\langle h_1|h_1\rangle\langle h_2|h_2\rangle}}.$$

$$\mathcal{F} = \max_{\lambda} \mathcal{M}[h_1, h_2(\lambda)], \quad \log_{10}\mathcal{B}_U^L = (\mathcal{F}_L^2 - \mathcal{F}_U^2)\frac{\mathrm{SNR}^2}{2}$$

Base Results: Unlensed Parameter Estimation Recoveries

$\mathcal{M}_B/\mathcal{M}_A \in \{0.5, 1, 2\}$, $\mathrm{SNR_B}/\mathrm{SNR_A} \in \{0.5, 1\}$, $\Delta t_c \in [-0.1, 0.1]s$

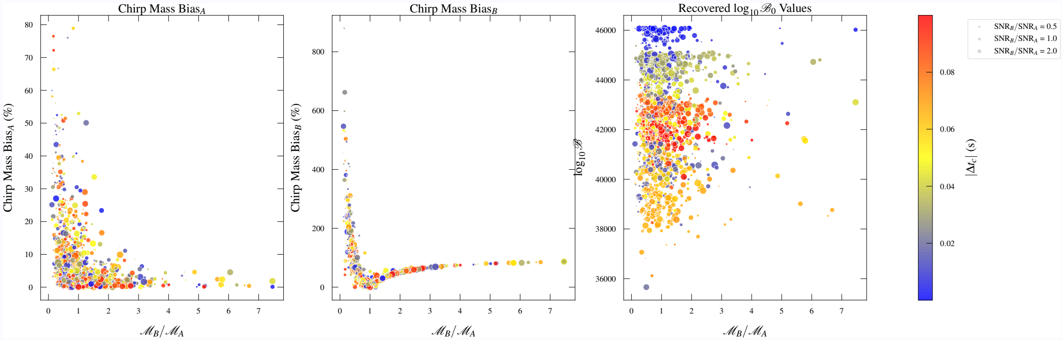
- Overlap induces significant parameter biases (e.g., in chirp mass).
- Higher-SNR signals dominate recovery.
- Bimodal distributions emerge in strongly overlapping cases.



$\mathcal{M}_B/\mathcal{M}_A = 2$, $\mathrm{SNR}_B/\mathrm{SNR}_A = 1$, $\Delta t_c = 0.01$ s.

Population Fitting Factor Results: Unlensed Signals

- Confirms the individual case dependencies and biases.
- Evident biases increase at stronger overlaps and comparable chirp mass, SNRs.

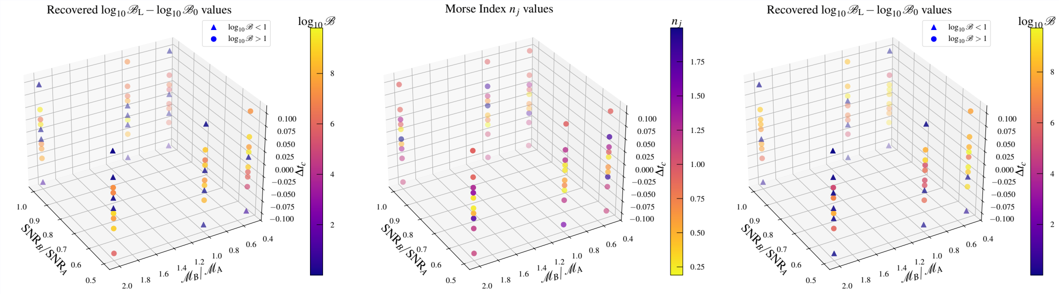


Type II Lensed Parameter Estimation Recoveries

$\mathcal{M}_B/\mathcal{M}_A \in \{0.5, 1, 2\},\ \mathrm{SNR_B}/\mathrm{SNR_A} \in \{0.5, 1\},\ \Delta t_c \in [-0.1, 0.1]s$

A : Fixed Morse phase shows distinct Bayes factor differences over the unlensed case.

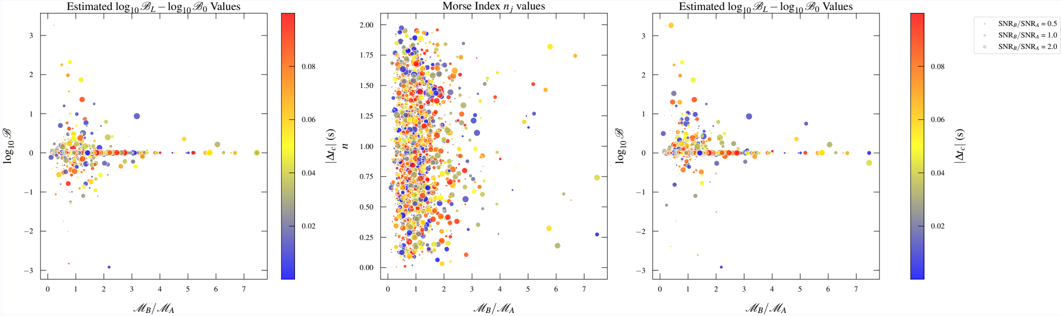
B,C : Allowing the Morse phase to vary improves lensing characterization.



A B C

Population Fitting Factor Results: Type II Lensed Signals

- A : Fixed Morse phase presents mild support for Type II lensing recovery, except at comparable chirp mass, SNRs.
- B, C : Allowing the Morse phase to vary shows greater support for strong lensing.

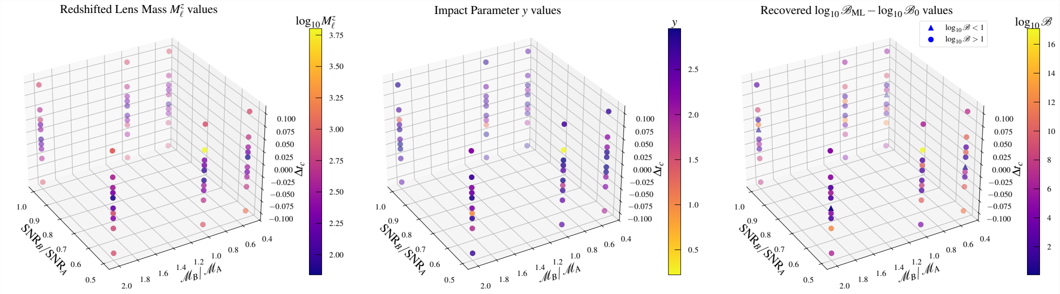


A B C

Microlensed Parameter Estimation Recoveries

$\mathcal{M}_B/\mathcal{M}_A \in \{0.5, 1, 2\},\ \mathrm{SNR_B}/\mathrm{SNR_A} \in \{0.5, 1\},\ \Delta t_c \in [-0.1, 0.1]s$

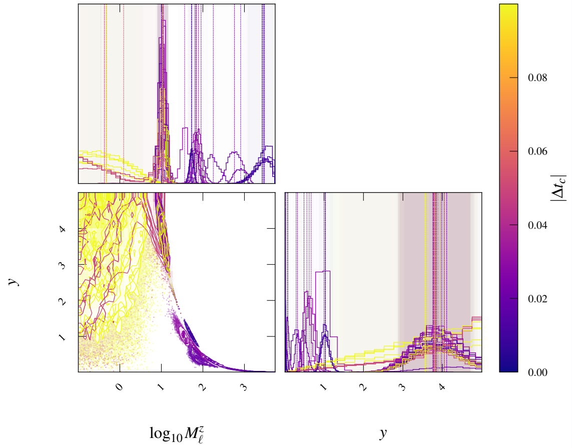
- Microlensed templates yield stronger support for strongly overlapping signals.
- Recovered lens parameters (M_ℓ^z, y) show dependencies with Δt_c.



Microlensing Parameter Estimation Posteriors

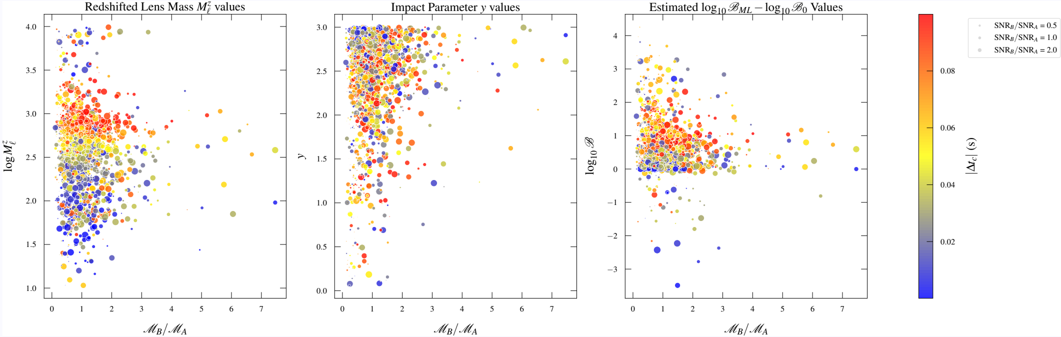
$\mathcal{M}_B/\mathcal{M}_A \in \{0.5, 1, 2\}$, $\mathrm{SNR_B}/\mathrm{SNR_A} \in \{0.5, 1\}$, $\Delta t_c \in [-0.1, 0.1]s$

- For cases with significant overlap, the microlensed recovery tends to favour **larger** lens masses and **smaller** impact parameters.
- Decrease in support for lensing at higher difference in coalescence times.



Population Fitting Factor Results: Microlensed Signals

- Significant support for microlensing at specific parameter regimes.
- Larger lens mass and smaller impact parameters recovered at significant chirp mass and SNR ratios.

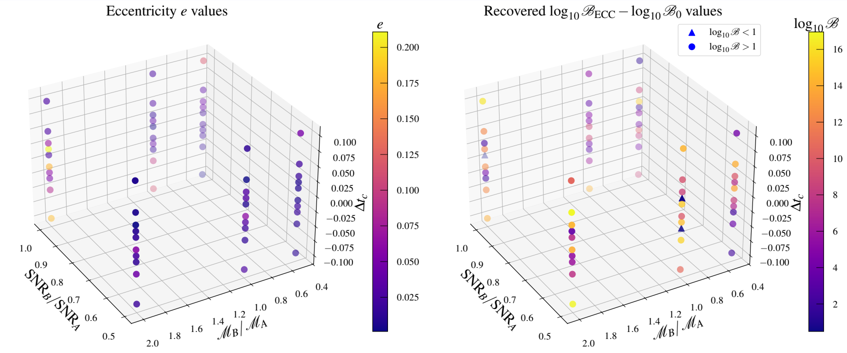


Conclusions

- Overlapping signals lead to significant biases in single signal unlensed parameter recovery.
- Further, we find that Type II lensing and microlensing signals can mimic overlapping effects, especially in the strong overlap regime.
- Advanced parameter estimation methods are essential to disentangle these effects.

Further Work

- Reanalyzing microlensing signals with the rectified Fourier convention.
- Future work will explore eccentric waveforms mimicking as a potential follow-up.



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- Relton, P. and Raymond, V., 2021. *Parameter estimation bias from overlapping binary black hole events in second generation interferometers.* Physical Review D, 104(8), p.084039.
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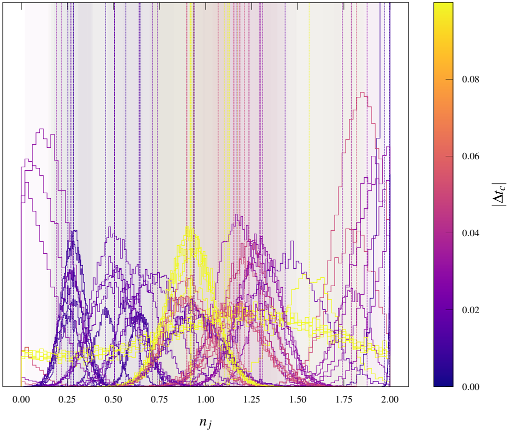
- Owen, B.J., 1996. *Search templates for gravitational waves from inspiraling binaries: Choice of template spacing.* Physical Review D, 53(12), p.6749.

Thank You!

Questions? Comments?

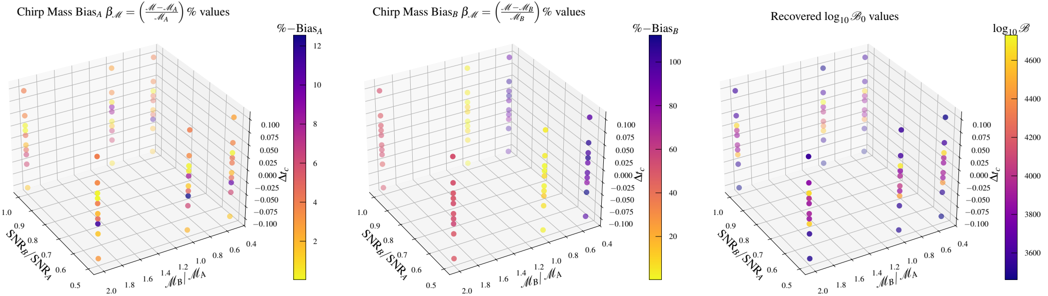
Type II Lensing Parameter Estimation Posteriors

$\mathcal{M}_B/\mathcal{M}_A \in \{0.5, 1, 2\},\ \mathrm{SNR_B}/\mathrm{SNR_A} \in \{0.5, 1\},\ \Delta t_c \in [-0.1, 0.1]s$



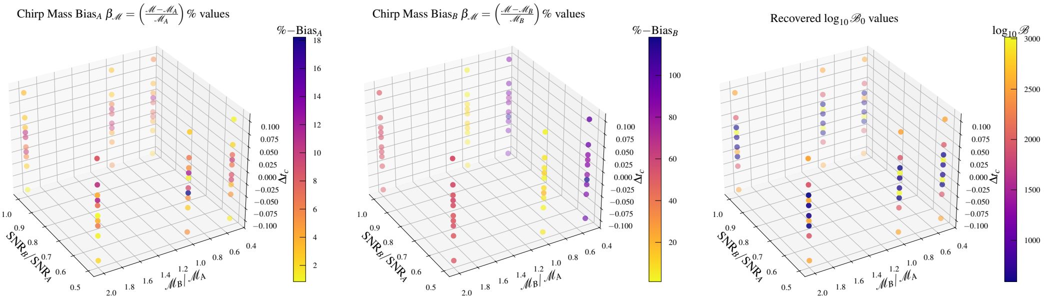
Unlensed Parameter Estimation Results

$\mathcal{M}_B/\mathcal{M}_A \in \{0.5, 1, 2\},\ \mathrm{SNR_B/SNR_A} \in \{0.5, 1\},\ \Delta t_c \in [-0.1, 0.1]s$



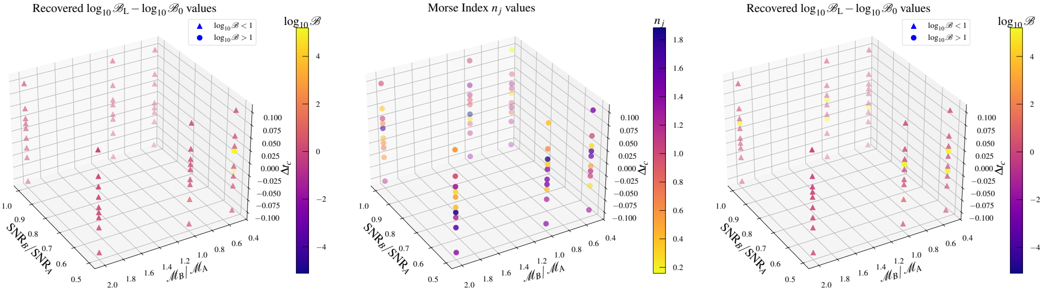
Unlensed Fitting Factor Results

$\mathcal{M}_B/\mathcal{M}_A \in \{0.5, 1, 2\}$, $\mathrm{SNR_B}/\mathrm{SNR_A} \in \{0.5, 1\}$, $\Delta t_c \in [-0.1, 0.1]s$



Type II Lensed Fitting Factor Results

$\mathcal{M}_B/\mathcal{M}_A \in \{0.5, 1, 2\}$, $\mathrm{SNR_B}/\mathrm{SNR_A} \in \{0.5, 1\}$, $\Delta t_c \in [-0.1, 0.1]s$



Microlensed Fitting Factor Results

$\mathcal{M}_B/\mathcal{M}_A \in \{0.5, 1, 2\},\ \mathrm{SNR_B}/\mathrm{SNR_A} \in \{0.5, 1\},\ \Delta t_c \in [-0.1, 0.1]s$