

# Eccentric non-spinning Ringdown

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**Nishkal Rao**<sup>1</sup>, Gregorio Carullo<sup>2</sup>,

<sup>1</sup>Indian Institute of Science Education and Research (IISER), Pune, India

<sup>2</sup>University of Birmingham, Edgbaston, Birmingham, United Kingdom

*[nishkal.rao@students.iiserpune.ac.in](mailto:nishkal.rao@students.iiserpune.ac.in)*

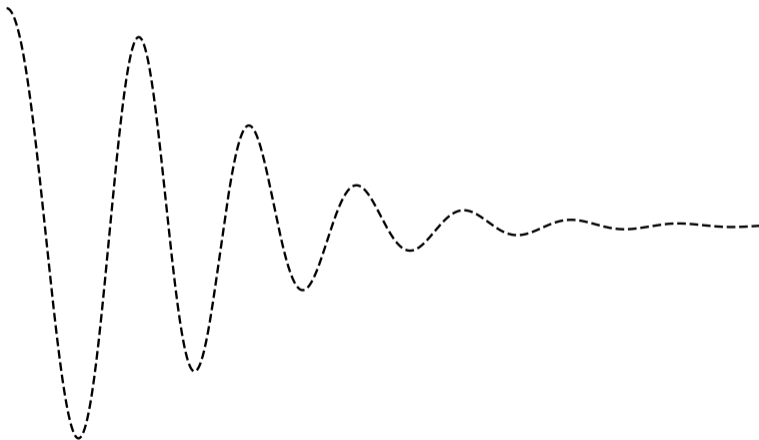
*[gregorio.carullo@ligo.org](mailto:gregorio.carullo@ligo.org)*

A whimsical illustration of a pond with ripples, trees, and a building in the background. The scene is bathed in a warm, golden light, suggesting a sunset or sunrise. In the foreground, there are silhouettes of people sitting on the grassy bank, looking towards the pond. The pond's surface is covered in concentric ripples, with two dark, round objects floating in the center. The background features a large, leafy tree on the right and a building with a steeple on the left. The overall atmosphere is peaceful and dreamlike.

# Eccentric Ringdown

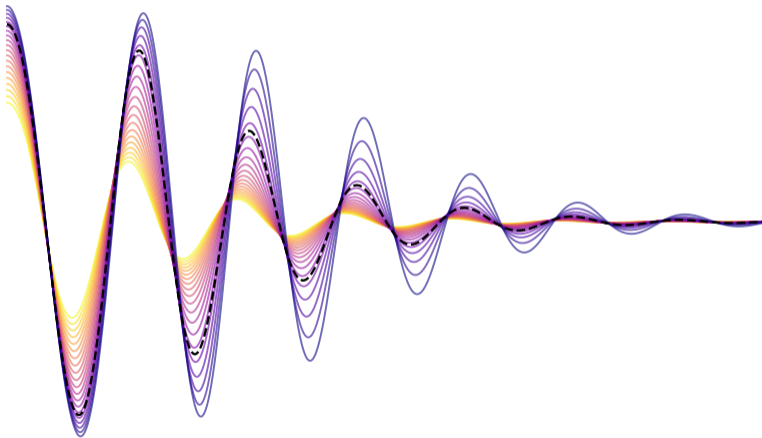
# Beyond Circularity

$$h_{22}(t) = A_{22}(t)e^{i(\omega_{220}t+\phi)}e^{-t/\tau_{220}}$$

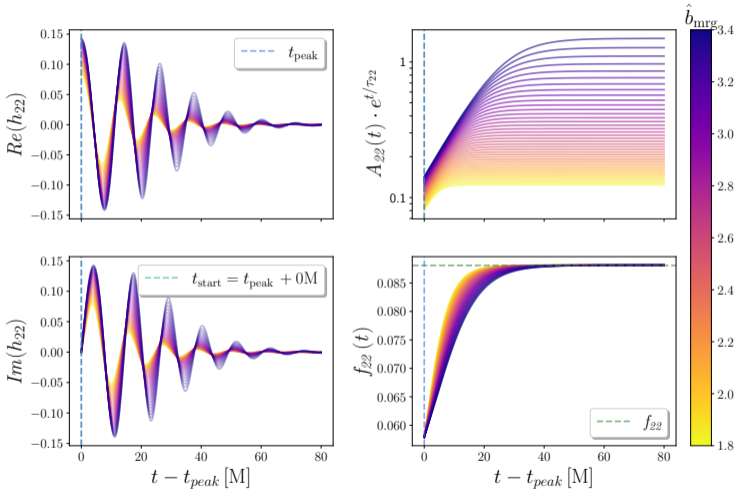


# Beyond Circularity

$$h_{22}(t) = A_{22}(t)e^{i(\omega_{220}t+\phi)}e^{-t/\tau_{220}}$$



# Waveform Modulations



Cubic Polynomial Global fit against  $\hat{b}_{\text{mrg}}$ .

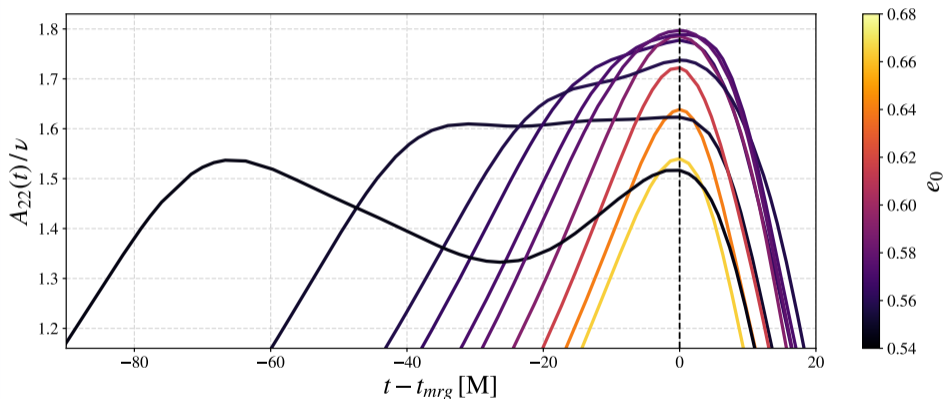


# Non-Circular Ringdown



# Eccentric Merger Data

Orbital-based parameterizations to dynamics-based, evaluated at the merger.



Carullo JCAP10 2024, 061.

# Generic Dynamic Parameterization

[Carullo, Gregorio, et al., 2024]

- From initial  $(E_0^{\text{ADM}}, J_0^{\text{ADM}})$ , at time  $t$ ,  $(E(t), J(t))$ , such that

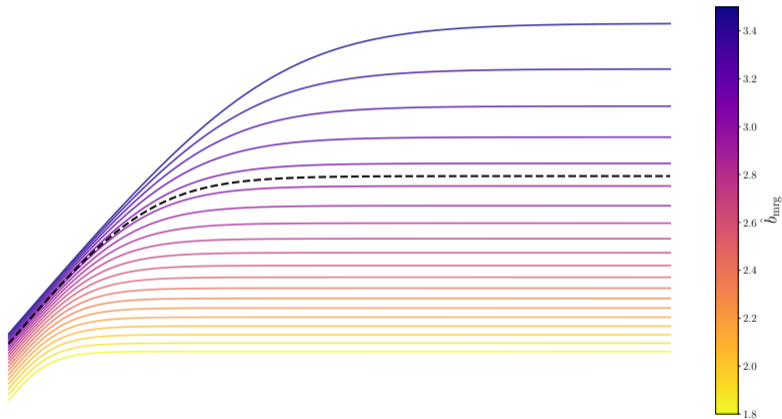
$$E(t) = E_0^{\text{ADM}} - \int_{t_0}^t dt' \dot{E}(t') \text{ and } J(t) = J_0^{\text{ADM}} - \int_{t_0}^t dt' \dot{J}(t'), \text{ where}$$

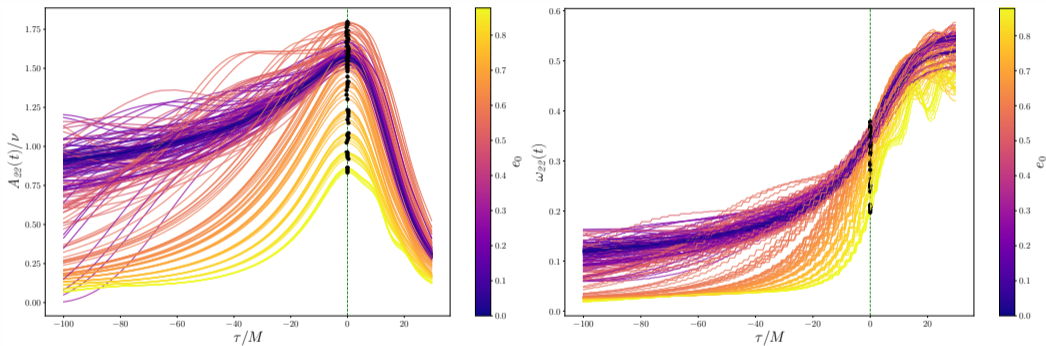
$$\dot{E} = \frac{1}{16\pi} \sum_{\ell=2}^{\ell_{\max}} \sum_{m=-\ell}^{\ell} |\dot{h}_{\ell m}|^2 \text{ and } \dot{J} = \frac{1}{16\pi} \sum_{\ell=2}^{\ell_{\max}} \sum_{m=-\ell}^{\ell} m \Im\{h_{\ell m} \dot{h}_{\ell m}^*\}.$$

- Merger quantities at  $t_{\text{mrg}}$  corresponding to the peak of the emission immediately before a ringdown begins.

# Amplitude Monotonicity

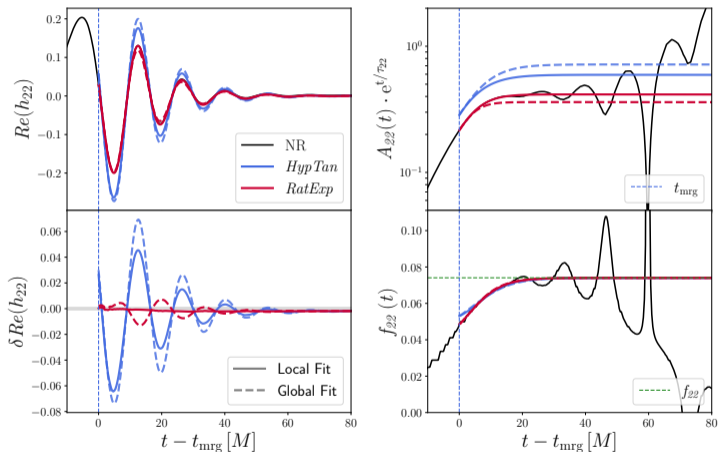
$$A_{22}(t)e^{t/\tau_{22}}$$



Defining  $t_{\text{mrg}}$ 

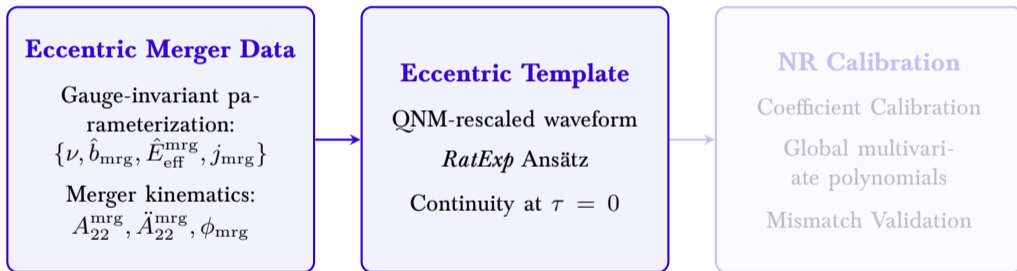
RIT Non-Spinning Simulations.

# Improvements



RIT:1479 -  $q \approx 0.33$ ,  $\chi_{1,2} = 0$ ,  $e_0 \approx 0.75$

# Non-Circular Ringdown



# Eccentric Template

[Albanesi, Simone, et al., 2023]

Adapting the non-circular post-merger ansatz to capture complex, highly-eccentric transients.

$$A_{\bar{h}}(\tau) = \left( \frac{c_1^A}{1 + \exp(-c_2^A \tau + c_3^A)} + c_4^A \right)^{\frac{1}{c_5^A}} \quad (1.1)$$

$$\phi_{\bar{h}}(\tau) = -c_1^\phi \ln \left( \frac{1 + c_3^\phi \exp(-c_2^\phi \tau) + c_4^\phi \exp(-2c_2^\phi \tau)}{1 + c_3^\phi + c_4^\phi} \right) \quad (1.2)$$

❶  $A_{\bar{h}}(\tau = 0) = A_{22}^{\text{mrg}}$  (NR).

❷  $\left. \frac{dA_{\bar{h}}}{d\tau} \right|_{\tau=0} = \dot{A}_{22}|_{t=t_0}$ .

❸  $\left. \frac{d^2 A_{\bar{h}}}{d\tau^2} \right|_{\tau=0} = \ddot{A}_{22}|_{t=t_0}$ .

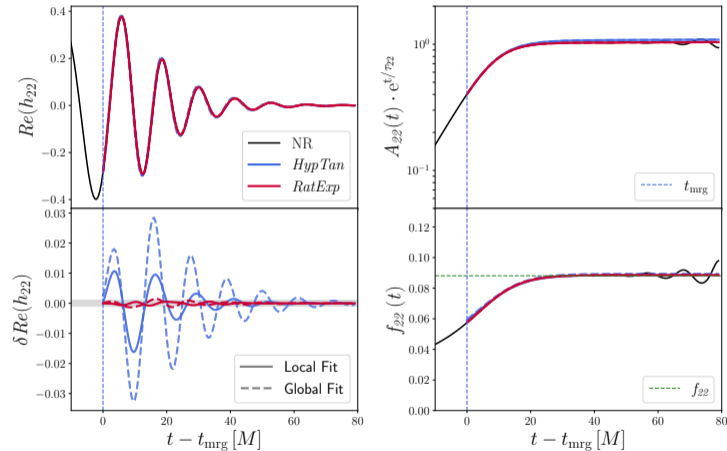
❹  $\left. \frac{d\phi_{\bar{h}}}{d\tau} \right|_{\tau=0} = \Delta\omega = \omega_1 - \mathcal{M}_{\text{BH}}\omega_{22}^{\text{mrg}}$ .

# Continuity Condition

- $c_1^A = \frac{c_5^A \alpha_1}{c_2^A} (A_{22}^{\text{mrg}})^{c_5^A} \exp(-c_3^A) (1 + \exp(c_3^A))^2$   
(from II)
- $c_2^A$  *free parameter*
- $c_3^A$  *free parameter*
- $c_4^A = (A_{22}^{\text{mrg}})^{c_5^A} - \frac{c_1^A}{1 + \exp(c_3^A)}$  (from I)
- $c_5^A = -\frac{\ddot{A}_{22}^{\text{mrg}}}{A_{22}^{\text{mrg}} \alpha_1^2} + \frac{c_2^A \exp(c_3^A) - 1}{\alpha_1 \exp(c_3^A) + 1}$  (from IV)

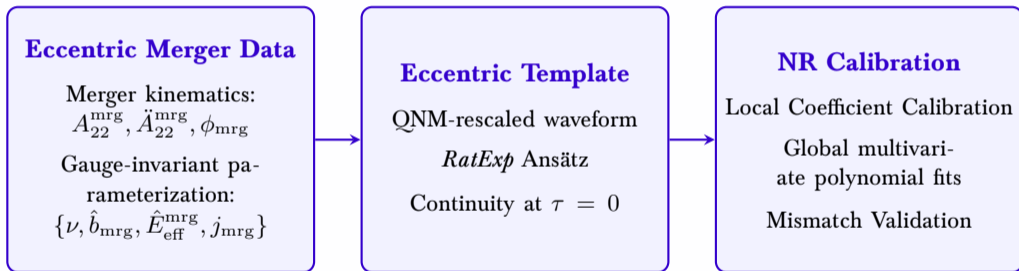
- $c_1^\phi = \frac{1 + c_3^\phi + c_4^\phi}{c_2^\phi (c_3^\phi + 2c_4^\phi)} \Delta\omega$  (from IV)
- $c_2^\phi$  *free parameter*
- $c_3^\phi$  *free parameter*
- $c_4^\phi$  *free parameter*

# Improvements



SXS:2527 -  $q \approx 1, \chi_{1,2} = 0, e_0 \approx 0.79$

# Non-Circular Ringdown



# NR Calibration

## Local Coefficient Extraction

### NR Data

233 Non-Spinning RIT Simulations

Filtered via balance laws ( $e_0 \leq 0.9$ )

### Local Parameter Estimation

$\{c_2^A, c_3^A, c_2^\phi, c_3^\phi, c_4^\phi\}$ : Bayesian extraction

bayRing package, median extraction from each simulation

# Global Fit

## Multivariate Polynomial

**Coefficients**  
 $\{c_2^A, c_3^A, c_2^\phi, c_3^\phi, c_4^\phi\}$

$$\tilde{Y} = Y_0 + \sum_{k=1}^d \sum_{|\alpha|=k} p_\alpha \prod_{i=1}^m Q_i^{\alpha_i}$$

**Mass Ratio  $\nu$**

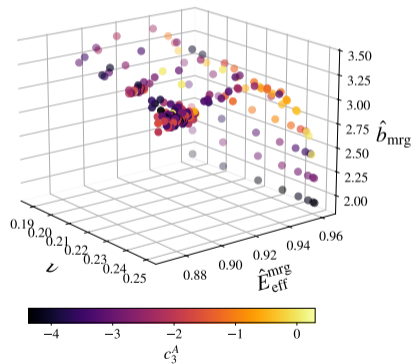
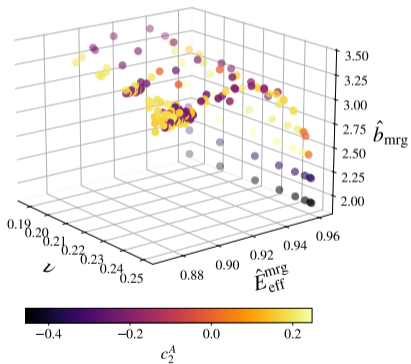
**Merger Dynamics**  
 $\{\hat{E}_{\text{eff}}^{\text{mrg}}, j_{\text{mrg}}, \hat{b}_{\text{mrg}}\}$



# Results

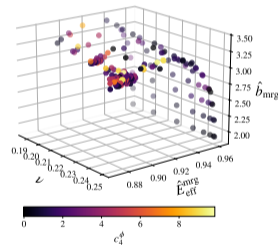
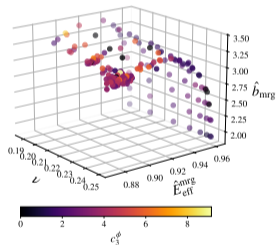
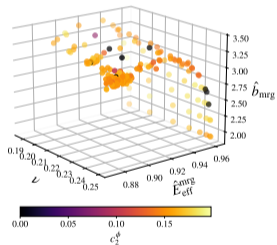
# Coefficient Calibration

Amplitude coefficients against 233 RIT simulations.

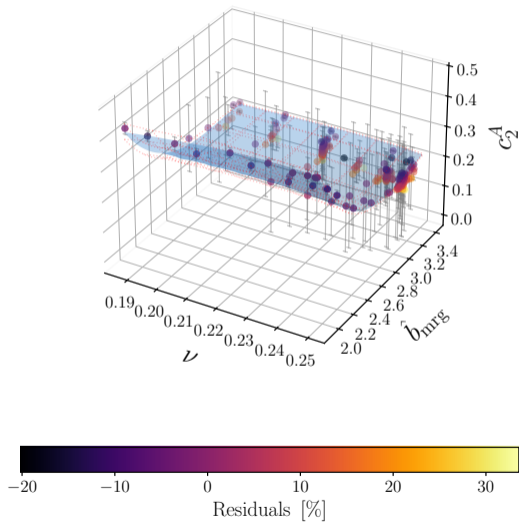


# Coefficient Calibration

Phase coefficients against 233 RIT simulations.

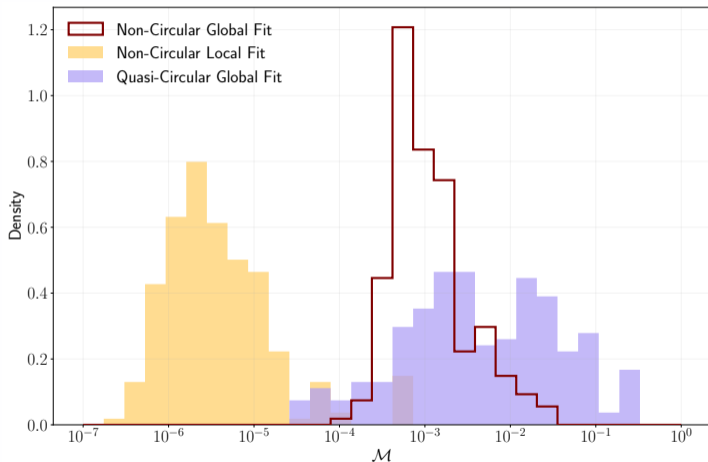


## Global Fits



# Mismatches

Cubic  $\{\nu, \hat{E}_{\text{eff}}^{\text{mrg}}, \hat{b}_{\text{mrg}}\}$  Global Fit









## Conclusions & Further Work

- Constructed an accurate numerically-informed post-merger waveform model for highly eccentric non-spinning binary black hole mergers (achieving mismatches around  $10^{-3}$ ).
- Extensions: Incorporation of intrinsic spin dynamics and higher-order waveform modes, and generalizing the framework to dynamically bound orbital captures.
- Seamless attachment to EOB waveforms for complete eccentric waveform generation and robust parameter estimation.

Thanks!

# References I

-  Nagar, Alessandro, et al. *Effective-one-body waveforms from dynamical captures in black hole binaries*. Physical Review D 103.6: 064013.
-  Buonanno, Alessandra, and Thibault Damour. *Effective one-body approach to general relativistic two-body dynamics* Physical Review D 59.8: 084006.
-  Damour, Thibault, and Alessandro Nagar. *New analytic representation of the ringdown waveform of coalescing spinning black hole binaries*. Physical Review D 90.2: 024054.
-  Carullo, Gregorio. *Ringdown amplitudes of nonspinning eccentric binaries*. Journal of Cosmology and Astroparticle Physics 2024.10: 061.
-  Albanesi, Simone, et al. *Faithful effective-one-body waveform of small-mass-ratio coalescing black hole binaries: The eccentric, nonspinning case*. Physical Review D 108.8: 084037.
-  Carullo, Gregorio, et al. *Unveiling the merger structure of black hole binaries in generic planar orbits*. Physical Review Letters 132.10: 101401.

# Appendix

# Ringdown Waveform model

Superposition of damped sinusoids and an exponentially decaying tail term.

$$\begin{aligned}
 h_{\ell m}(t) = & \sum_{\ell' mn}^{\infty} \left[ A_{\ell n}^+ \exp\{i(\omega_{\ell' mn}^+(t - t_{\text{ref}}) + \phi_{\ell' mn}^+)\} + \right. \\
 & \left. A_{\ell' mn}^- \exp\{i(\omega_{\ell' mn}^-(t - t_{\text{ref}}) + \phi_{\ell' mn}^-)\} \right] \theta(t - t_{\text{start}}^{\text{ref}}) + \\
 & A_{\ell m}^T \exp\{i\phi_{\ell m}^T\} (t - t_{\text{ref}})^{p_{\ell m}^T} \theta(t - t_{\text{start}}^{\text{tail}}).
 \end{aligned}$$

# Quasi-Circular Model

$$A_{\bar{h}}(\tau) = c_1^A \tanh(c_2^A \tau + c_3^A) + c_4^A$$

$$\phi_{\bar{h}}(\tau) = -c_1^\phi \ln \left( \frac{1 + c_3^\phi \exp(-c_2^\phi \tau) + c_4^\phi \exp(-2c_2^\phi \tau)}{1 + c_3^\phi + c_4^\phi} \right)$$

Ⓘ  $A_{\bar{h}}(\tau = 0) = A_{22}^{\text{mrg}}$  (NR).

Ⓜ  $\left. \frac{dA_{\bar{h}}}{d\tau} \right|_{\tau=0} = \dot{A}_{22}|_{t=t_0}$ .

Ⓝ  $c_2^A = \frac{\alpha_2 - \alpha_1}{2}$ .

Ⓧ  $\left. \frac{d\phi_{\bar{h}}}{d\tau} \right|_{\tau=0} = \Delta\omega = \omega_1 - \mathcal{M}_{\text{BH}} \omega_{22}^{\text{mrg}}$ .

Ⓨ  $c_2^\phi = \alpha_2 - \alpha_1$ .

# Quasi-Circular Model

From the continuity conditions,

- $c_1^A = A_{22}^{\text{mrg}} \alpha_1 \frac{\cosh^2 c_3^A}{c_2^A}$  (from II, III)
- $c_2^A = \frac{\alpha_2 - \alpha_1}{2}$  (from III)
- $c_3^A$  *free parameter*
- $c_4^A = A_{22}^{\text{mrg}} - c_1^A \tanh(c_3^A)$  (from I)

- $c_1^\phi = \frac{(\omega_1 - \mathcal{M}_{\text{BH}} \omega_{22}^{\text{mrg}})(1 + c_3^\phi + c_4^\phi)}{c_2^\phi (c_3^\phi + 2c_4^\phi)}$  (from IV)
- $c_2^\phi = \alpha_2 - \alpha_1$  (from IV)
- $c_3^\phi$  *free parameter*
- $c_4^\phi$  *free parameter*