Black Hole Decoherences and Superpositions

Presentation

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Agenda

The Problem

[Belenchia, Alessio, et al., 2018]

Gravitational field associated with a quantum source.

Superposition of gravitational fields.

Entanglement due to gravitational interaction

[Bose, Sougato, et al., 2017]

- Stern-Gerlach (SG) interferometry over gravitationally entangled spin systems.
- Two test masses initially spatially localized $|L\rangle$ and $|R\rangle$.
- SG: Initial $|C_j\rangle \to |L, \uparrow\rangle_j + |R, \downarrow\rangle_j$.
- Evolution under mutual gravitational interaction $\implies$ Entanglement[1].
- Gravitational mediated through h_{00} perturbation as a *quantum coherent mediator*.

[1] cannot be created by local operations and classical communication

Entanglement due to gravitational interaction

[Bose, Sougato, et al., 2017]

Time evolution:

$$|\Psi(t=0)\rangle_{12} = \frac{1}{\sqrt{2}}(|L_1\rangle + |R_1\rangle) \otimes \frac{1}{\sqrt{2}}(|L_2\rangle + |R_2\rangle)$$

$$|\Psi(t=\tau)\rangle_{12} = \frac{e^{i\phi}}{\sqrt{2}}\{|L_1\rangle \otimes \frac{1}{\sqrt{2}}(|L_2\rangle + e^{i\Delta\phi_{LR}}|R_2\rangle)$$

$$+ |R_1\rangle \otimes \frac{1}{\sqrt{2}}(e^{i\Delta\phi_{RL}}|L_2\rangle + |R_2\rangle)\}$$

Difference in phases due to gravitational energy differences.
Entanglement if $\Delta\phi_{LR} + \Delta\phi_{RL} \neq 2n\pi$.

Entanglement due to gravitational interaction

[Bose, Sougato, et al., 2017]

Resolution:

- Weak gravity in the non-relativistic limit determined by the scalar constraint of General Relativity.
- Relevant gravitational degrees of freedom in the proposed experiments are pure-gauge, with no physical content.
- True degrees of freedom are transverse-traceless gravitational waves through linearized perturbations.
- Newtonian interaction term is through pure-gauge component Φ, by $\nabla^2\Phi = -4\pi G\rho$, representing the scalar constraint.

gedankenexperiment

[Belenchia, Alessio, et al., 2018]

- A: Adiabatic separation of $|L\rangle_A$ and $|R\rangle_A$ from a SG experiment, separated by d, minimal radiation emitted. Coupled with the spins along z direction.
  B: $t = 0$, choose to release a trapped particle (with a confining potential) or not.
- Correlated center of mass of particle B with A's location, depending on the amplitudes.
- B: After T_B time, δx displacement of the center of mass is measured. For large δx, orthogonality implies maximal correlation.
- A: $t = 0$, sends it through a reversing SG experiment to observe interference. Completes process in T_A time. Measures spin along x to analyze coherence (Ensemble averages).

gedankenexperiment

[Belenchia, Alessio, et al., 2018, Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2021]

- Spacelike separation: If B can acquire which path information in a time $T_B < D$ and A recombines the superposition in a time $T_A < D$ without emitting radiation, then inconsistencies with causality or complementarity arise.
- Complementarity: Partial decoherence of A, fail to obtain pure state if B releases. Implies violation of causality; Superluminal communication by noting the A's final state.

Electromagnetic Analysis

[Belenchia, Alessio, et al., 2018]

- $t = 0,\ |\Psi\rangle = \frac{1}{\sqrt{2}}\Big[\underbrace{|L\rangle_A}_{\text{SG Left}}\ |\downarrow\rangle_A\ \underbrace{|\alpha_L\rangle_F}_{\text{EM State}} + \underbrace{|R\rangle_A}_{\text{SG Right}}\ |\uparrow\rangle_A\ \underbrace{|\alpha_R\rangle_F}_{\text{EM State}}\Big] \otimes |\psi_0\rangle_B.$
- This equation represents a notional form, since $|L/R; \alpha_{L/R}\rangle_A$ can not be decomposed. Note that the decoherence due to the electromagnetic field is false and can be recovered on recombination.
- Causal separation prevents the local electromagnetic field from causing decoherence for $T_A < D$ and $T_B < D$.

Vaccuum Fluctuations

[Belenchia, Alessio, et al., 2018]

- Electric field averaged over a spacetime region R of the order $\Delta E \propto \frac{1}{R^2}$. Relevant for a wordline of timescale R.
- $\dot{x} = \int dt\ \frac{qE}{m}$, hence $\Delta x = \int dt\ \dot{x} \sim \frac{q}{M}$, independent of R. Localization around a charge-radius. Stringent limit over Compton limit.
- Displacement non localisation if $\delta x > \frac{q_B}{m_B}$ of B. Constrains the time T_B.

Vaccuum Fluctuations

[Belenchia, Alessio, et al., 2018]

- Estimating δx, we note the detection of the electric field induced by the effective dipole moment $\mathcal{D}_A = q_A d$.
- Since $E \sim \frac{\mathcal{D}_A}{D^3}$, for T_B time, $\delta x \sim \frac{q_B E}{m_B} T_B^2 = \frac{q_B}{m_B} \frac{\mathcal{D}_A}{D^3} T_B^2$ displacement. Since $\delta x > \frac{q_B}{m_B}$, we have $\frac{\mathcal{D}_A}{D^3} T_B^2 > 1$.

Quantized Electromagnetic Radiation

[Belenchia, Alessio, et al., 2018]

- Effective dipole moment $\mathcal{D}_A$ has to vanish on reverse SG. Sufficiently slow recombination must be ensured.
- $|\alpha_L - \alpha_R\rangle_F \sim \mathcal{O}(1)$ would make states orthogonal and decoherence.
- The difference in charge-current corresponds to effective dipole $\mathcal{D}_A(t)$, whose energy flux $\sim (\ddot{\mathcal{D}}_A)^2 \sim \left(\frac{\mathcal{D}_A}{T_A^2}\right)^2$ since it changes over a characteristic time T_A.

Quantized Electromagnetic Radiation

[Belenchia, Alessio, et al., 2018, Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2021]

- By Larmor formula, total energy over T_A time, $\mathcal{E} \sim \int_0^{T_A} dt\ (\ddot{\mathcal{D}}_A)^2 \sim \left(\frac{\mathcal{D}_A}{T_A^2}\right)^2 T_A$.
- This energy is realized in quantum theory as photons with frequency $\sim \frac{1}{T_A}$.
- Thereby, the number of photons of the order $\mathcal{N} \sim \left(\frac{\mathcal{D}_A}{T_A}\right)^2$, implying $\mathcal{D}_A < T_A$ for avoiding entanglement.

Spacelike Separation $(T_A, T_B < D)$

[Belenchia, Alessio, et al., 2018]

- If $\mathcal{D}_A < T_A$
  - A closes her superposition without emitting entangling radiation.
  - Since $\mathcal{D}_A < T_A < D$, B cannot obtain which-path information in time $T_B < D$.
  - A successfully recoheres her particle; B does nothing.
- If $\mathcal{D}_A > T_A$
  - A necessarily emits entangling radiation, so her recoherence experiment fails.
  - B's particle gains which-path information by entangling with the already-radiated field.
  - B remains an innocent bystander in the decoherence.

Beyond $T_A < D$

[Belenchia, Alessio, et al., 2018]

- Consider dropping $T_A < D$, and suppose $\mathcal{D}_A > D$. Then:
  - If A takes $T_A > \mathcal{D}_A$, no entangling radiation is emitted.
  - Without B releasing his particle: A successfully recoheres.
  - With B releasing: his particle becomes entangled with A's, and her recoherence fails.
  - Since $T_A > \mathcal{D}_A > D$, no causality issues arise.
- In this case, what would be false decoherence (from A's radiation) becomes true decoherence if B acts.

Collecting the Radiation

[Belenchia, Alessio, et al., 2018]

- For $D < \mathcal{D}_A$, B can gain which-path information unless A collects the radiation.
- A's options for a spherical mirror:
  - I Mirror present throughout the experiment.
  - II Mirror erected over a time $T_M < D$ starting near $t = 0$.
- Mirror placement:
  - a $R_M < D$: (Case I(a)) Successful recoherence; B is shielded.
  - b $R_M > D$: (Cases I(b) and II(b)) B obtains which-path information, causing decoherence.
- In case II(a), the mirror's erection ($T_M < D < \mathcal{D}_A$) creates a time-changing dipole, emitting entangling photons much like the $T_A < D < \mathcal{D}_A$ case.

Summary of Consistency

[Belenchia, Alessio, et al., 2018]

- Compatible with causality and complementarity.
- Vacuum fluctuations of the electromagnetic field prevent B from obtaining which-path information too quickly in the case $\mathcal{D}_A < D$.
- Quantized radiation ensures that if $\mathcal{D}_A > D$, the emitted photons cause true decoherence.
- Without these, one could violate causality (if B influences A's state) or complementarity (if B doesn't).

Analysis with gravitational interaction

[Belenchia, Alessio, et al., 2018]

- Implications of vacuum fluctuations: The absence of background structure in general relativity implies that the location of a particle is not a well-defined concept.
- Relative location: For bodies separated by R, the magnitude of the vacuum fluctuations of the Riemann curvature tensor $\mathcal{R}$ is estimated. In quantum gravity, the typical fluctuation in the metric is set by the Planck length l_P.

Analysis with gravitational interaction

[Belenchia, Alessio, et al., 2018]

- We may argue dimensionally that if a metric fluctuation is of order $\Delta g \sim \frac{l_P}{R}$, then the curvature (being $\sim \partial^2 g$) scales as $\Delta \mathcal{R} \sim \frac{l_P}{R^3}$.
- From the geodesic equation integration, we have $\Delta x \sim l_P$, independently of R. Differences in the gravitational fields resulting from the different components of the wavefunction must be large enough to produce a displacement $\delta x > l_P$.

Careful and Detailed investigation

[Belenchia, Alessio, et al., 2018]

- Conservation of stress-energy in a (nearly) flat spacetime implies that the centre of mass of the total system moves on an inertial trajectory.
- Entanglement with the laboratory resulting in false decoherences:
$$|\Psi\rangle = \frac{1}{\sqrt{2}}\Big[\underbrace{|L\rangle_A}_{\text{SG Left}} |\downarrow\rangle_A \underbrace{|\alpha_L\rangle_F}_{\text{GR State}} \underbrace{|\beta_L\rangle_A}_{\text{Lab State}} + \underbrace{|R\rangle_A}_{\text{SG Right}} |\uparrow\rangle_A \underbrace{|\alpha_R\rangle_F}_{\text{GR State}} \underbrace{|\beta_R\rangle_A}_{\text{Lab State}}\Big] \otimes |\psi_0\rangle_B.$$
- The states $|L\rangle_A|\beta_L\rangle_A$ and $|R\rangle_A|\beta\rangle_A$ have the same centre of mass, vanishing the effective dipole moment.

Careful and Detailed investigation

[Belenchia, Alessio, et al., 2018]

- Effective mass quadropole $\mathcal{Q}_A$ implies seperation of B $\delta x \sim \frac{\mathcal{Q}_A}{D^4} T_B^2$ since the gravitational field associated is $F \sim \frac{\mathcal{Q}_A}{D^4}$.
- Thus, non localisation and probing measurement possible if $\delta x \sim \frac{\mathcal{Q}_A}{D^4} T_B^2 > l_P \equiv 1$.
- Given the quadrupole interaction, the number of photons radiated is of the order $\mathcal{N} \sim \left(\frac{\mathcal{Q}_A}{T_A^2}\right)^2$, implying $\mathcal{Q}_A < T_A^2$ for avoiding entanglement.

Case Analysis

[Belenchia, Alessio, et al., 2018]

- For $T_A < D$ and $T_B < D$,
  1. $\mathcal{Q}_A < T_A^2$
     - A avoids emitting radiation, B cannot obtain which-path information in $T_B < D$.
     - A successfully recoheres her particle with no inconsistencies.
  2. $\mathcal{Q}_A > T_A^2$
     - A necessarily emits entangling gravitational radiation and decoheres.
     - B can obtain which-path information in $T_B < D$, but only by transferring the entanglement already present in the gravitational field.
     - B is an innocent bystander in A's decoherence.
- Other cases follow similarly to the electromagnetic version.

Conclusive Remarks

[Belenchia, Alessio, et al., 2018]

To avoid contradictions with complementarity or causality, quantum gravity must have fundamental features of a quantum field theory at low energies, specifically the quantization of gravitational radiation (which decoheres A's particle without the presence of B) and local vacuum fluctuations (which limits B's ability to measure the position of A's particle).

Decoherence of A

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2021]

- During the recombination, there is no way to meaningfully separate $|\alpha_L\rangle$ and $|\alpha_R\rangle$ into a Coulomb part and a Radiation part.
- But at asymptotic late times, the electromagnetic field naturally decomposes into a radiation field that propagates to null infinity and a Coulomb field that follows A to timelike infinity. $|\Psi\rangle = \frac{1}{\sqrt{2}}\Big[|L;\downarrow\rangle_{i^+}|\alpha_L\rangle_{\mathscr{I}^+} + |R;\uparrow\rangle_{i^+}|\alpha_R\rangle_{\mathscr{I}^+}\Big] \otimes |\beta_0\rangle_B$, where $|L\rangle_{i^+} = |R\rangle_{i^+}$ after recombination.

Decoherence of A

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2021]

- To extend it backwards in time, consider the point of recombination and an arbitrary Cauchy surface Σ through it. Assuming inertial motion, after recombination, we note the causal future would correspond to the Coulomb field.
- For these well-defined states, the degree of decoherence for A is given by $\mathscr{D}_A = 1 - \left|\langle \alpha_L | \alpha_R \rangle_{\mathscr{I}^+}\right|$, which relates to the number of entangling photons emitted $\langle \alpha_L | \alpha_R \rangle_{\mathscr{I}^+} \sim \langle \alpha_L - \alpha_R | 0 \rangle_{\mathscr{I}^+}$.

Decoherence of A

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2021]

- We extend the Coulomb field to $\mathscr{I}^+(\Sigma)$ and subtract this from the local electromagnetic field to preserve a source-free field, corresponding to well-defined states $|\alpha_L\rangle_\Sigma$ and $|\alpha_R\rangle_\Sigma$ on Σ.
- Since we evolve unitarily from $|\alpha_{L/R}\rangle_\Sigma \to |\alpha_{L/R}\rangle_{\mathscr{I}^+}$, we preserve the inner products to have $\mathscr{D}_A = 1 - \left|\langle\alpha_L|\alpha_R\rangle_\Sigma\right|$. Minimal decoherence can be achieved by doing the recombination adiabatically.

Decoherence of B

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2021]

- We allow B to make any field measurement and not constrain displacement localization, which can be improved with multiple measurements.
- The electromagnetic field at null infinity is $|0\rangle_{\mathscr{I}^+}$, independent of the system. The final state $|\Psi\rangle = \frac{1}{\sqrt{2}}\Big[|L;\downarrow\rangle_{i^+} \otimes |\beta_L\rangle_{i^+} + |R;\uparrow\rangle_{i^+} \otimes |\beta_R\rangle_{i^+}\Big]|0\rangle_{\mathscr{I}^+}$.
- The decoherence represents the failure to coincide $\mathscr{D}_B = 1 - \left|\langle\beta_L|\beta_R\rangle_{i^+}\right|$, which is equivalent to $\mathscr{D}_B = 1 - \left|\langle\beta_L|\beta_R\rangle_{T_B}\right|$.

Re-Analysis of the Experiment

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2021]

- At the time represented by a Cauchy surface Σ_1 passing through the point of recombination, but lies to the past of B's measurements, we have the correlation as earlier with $\mathscr{D}_A = 1 - \left|\langle\alpha_L|\alpha_R\rangle_{\Sigma_1}\right|$, without B decohering.
- Now consider a time represented by another Cauchy surface Σ_2 before the recombination of A, but after the measurement characterization of B, leading to the decoherence of A with $\mathscr{D}_B = 1 - \left|\langle\beta_L|\beta_R\rangle_{\Sigma_2}\right|$ independent of A.
- Paradoxical nature if $\mathscr{D}_B > \mathscr{D}_A$, resulting in $\left|\langle\beta_L|\beta_R\rangle_{\Sigma_1}\right| < \left|\langle\alpha_L|\alpha_R\rangle_{\Sigma_1}\right|$.

Re-Analysis of the Experiment

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2021]

- We show, such a contradiction can't happen by considering the state at Σ_1, post recombination of A, but before B starts measuring to be $|\Psi\rangle = \frac{1}{\sqrt{2}}\Big[|L;\downarrow\rangle|\alpha_L\rangle_{\Sigma_1} + |R;\uparrow\rangle|\alpha_R\rangle_{\Sigma_1}\Big] \otimes |\beta_0\rangle$.
- After evolution of measurement of B at a Cauchy surface Σ_3, we have $|\Psi'\rangle = \frac{1}{\sqrt{2}}\Big[|L;\downarrow\rangle|\alpha'_L\rangle_{\Sigma_3}|\beta_L\rangle_{\Sigma_3} + |R;\uparrow\rangle|\alpha'_R\rangle_{\Sigma_3}|\beta_R\rangle_{\Sigma_3}\Big]$.
- The post-measurement states of the radiation depend on the interaction with B, but the joint state $|\alpha_{L/R}\rangle|\beta_{L/R}\rangle$ evolves unitarily, thereby, we have $\langle\beta_L|\beta_R\rangle_{\Sigma_3}\langle\alpha'_L|\alpha'_R\rangle_{\Sigma_3} = \langle\beta_0|\beta_0\rangle\langle\alpha_L|\alpha_R\rangle_{\Sigma_1} = \langle\alpha_L|\alpha_R\rangle_{\Sigma_1}$.

Re-Analysis of the Experiment

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2021]

- Hence, $\left|\langle\beta_L|\beta_R\rangle_{\Sigma_1}\right| \geq \left|\langle\alpha_L|\alpha_R\rangle_{\Sigma_1}\right|$ for any field measurement of B. There is no violation of causality or complementarity.
- Noting the amplitude of emitted radiation in a spacelike separation is a characteristic feature of quantum field theory. The observer in the spacelike-separated region cannot tell whether observing a photon or a vacuum fluctuation.
- Indistinguishability of superposition of the Coulomb fields of A to the past of time Σ_2, with the single Coulomb field of recombined particle together with free radiation to the future of Σ_1.

Gravitational Analysis of the Experiment

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2021]

- Direct relationship between Newtonian entanglement and the existence of gravitons.
- A became entangled with on-shell gravitons emitted during the recombination process, and B's apparatus then interacted with these gravitons, transferring some of the entanglement. Evident if B does not measure.
- The Newtonian gravitational field of A mediated an entanglement of B's apparatus with A. This is shown when A recombines later adiabatically.

Enter Black Holes!

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2022]

- Black hole in the vicinity of A. Ensure it doesn't fall in! Stationery maintenance without emitting radiation. Symmetry of the gravitational and spacetime curvature effects on the decoherence.
- Propagation of electromagnetic radiation field through the event horizon and null infinity.

Raditaion field propagation

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2022]

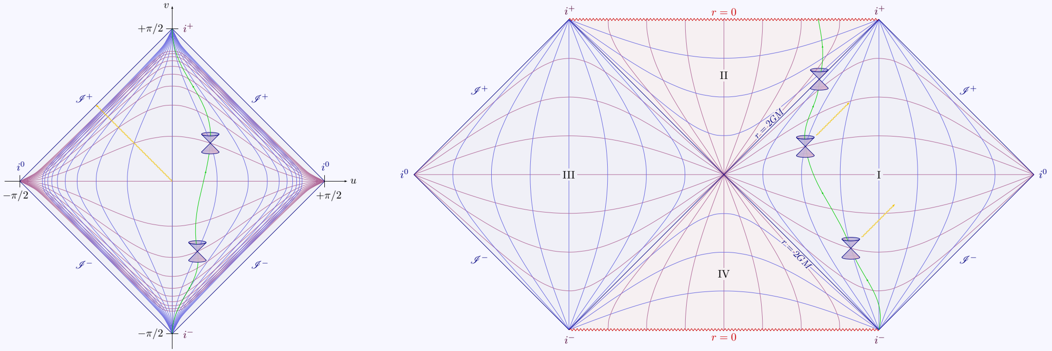

Figure: Penrose diagrams for Minkowski spacetime and Extended Schwarzschild black hole.

Decoherences on the horizon

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2022]

- At asymptotic late times, the final state is
$|\Psi\rangle = \frac{1}{\sqrt{2}}\Big[|L;\downarrow\rangle_{i^+}|\alpha_L\rangle_{\mathscr{I}^+}|\mathscr{A}_L\rangle_{\mathscr{H}^+} + |R;\uparrow\rangle_{i^+}|\alpha_R\rangle_{\mathscr{I}^+}|\mathscr{A}_R\rangle_{\mathscr{H}^+}\Big] \otimes |\beta_0\rangle_B$.
- Effective decoherence $\mathscr{D}_A = 1 - \big|\langle\alpha_L|\alpha_R\rangle_{\mathscr{I}^+}\langle\mathscr{A}_L|\mathscr{A}_R\rangle_{\mathscr{H}^+}\big|$. If adiabatic recombination ensuring negligible radiation emitted, we have the estimate $|\alpha_L\rangle_{\mathscr{I}^+} \approx |\alpha_R\rangle_{\mathscr{I}^+} \approx |0\rangle_{\mathscr{I}^+}$, resulting in $\mathscr{D}_A = 1 - \big|\langle\mathscr{A}_L|\mathscr{A}_R\rangle_{\mathscr{H}^+}\big|$.

Schwarzschild Electrodynamics

[Cohen, Jeffrey M., and Robert M. Wald., 1971]

- Maxwell's equation in curved spacetime:

$$4\pi j^{\mu} = F^{\nu\mu}_{;\nu} = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^{\nu}}[\sqrt{-g}F^{\nu\mu}] = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^{\nu}}\left[\sqrt{-g}g^{\nu\alpha}g^{\mu\beta}(A_{\beta,\alpha} - A_{\alpha,\beta})\right]$$

- Since the field of the point charge must be static and axially symmetric, the components of the electromagnetic field will not be a function of time or φ. Since the spacelike components of the current vanish, we take $A_i = 0$.

Schwarzschild Electrodynamics

[Cohen, Jeffrey M., and Robert M. Wald., 1971]

- Reducing, we have

$$4\pi j^0 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial A_0}{\partial r}\right) + \frac{1}{1-\frac{2M}{r}}\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial A_0}{\partial\theta}\right)$$

- We expand the angular part in terms of Legendre polynomials in the source-free regions where $j^0 = 0$. We solve the radial part to obtain independent solutions and their combinations.

Schwarzschild Electrodynamics

[Cohen, Jeffrey M., and Robert M. Wald., 1971,
Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2022]

- We obtain radial electrostatic fields for the horizon at $r \sim 2M$, but no net flux enters the black hole since as many flux lines exit the black hole from the side opposite the charge as they enter from the side near the charge.
- The dominant contribution to the field comes from the $\ell = 0$ term because the coefficients of the higher multipole terms vanish.
- Non-vanishing component on the horizon is $E_r = F_{\mu\nu} s^\mu n^\nu$, where $n^\mu = \left(\frac{\partial}{\partial V}\right)^\mu$ denotes the affinely parametrized null normal to the horizon and s^μ is the unique past-directed radial null vector satisfying $n^\mu s_\mu = 1$.

Schwarzschild Electrodynamics

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2022]

- Electromagnetic radiation dual to the null normal E_Ω characterized by angular components, is the pullback of $E_\mu = F_{\mu\nu}n^\nu$ to the horizon.
- For static point charge, $E_\Omega = 0$ and no radiation through the horizon. However $E_r \neq 0$ on $\mathscr{H}^+$.
- Quasi-static movement of the point charge results in $\mathcal{D}^\Omega E_\Omega = -\partial_V E_r$ by Maxwell's equations, resulting in some radiation through the horizon.

Schwarzschild Electrodynamics

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2022]

- Further, $\int \mathcal{D}^{\Omega} E_{\Omega} dV = \Delta E_r$, hence $\int E_{\Omega} dV$ is constrained by the initial and final values of E_r and is independent of how slowly the charge is moved.
- There is necessarily some radiation that crosses the horizon of the black hole due to the displacement of the charge.
- If the point charge is moved very slowly, the total energy radiated into the black hole $\propto \int E^{\Omega} E_{\Omega} r^2 d\Sigma V dV$ can be made arbitrarily small.

Quantum state of electromagnetic radiation

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2022, Gerard, Christian., 2018]

- For an unperturbed black hole formed by gravitational collapse, the state of the electromagnetic field on the horizon of the black hole is described by the Unruh vacuum.
- For low-frequency phenomena $\omega \ll \frac{c^3}{GM}$, in which case the Unruh and Hartle-Hawking vacua near the horizon are essentially indistinguishable.
- In the Fock space associated with the Hartle-Hawking vacuum, a particle corresponds to a solution that is purely positive frequency with respect to the affine parameter on the horizon.

Quantum state of electromagnetic radiation

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2022]

- Perturbing the black hole by a classical charge-current source of the quantum electromagnetic field.
- The quantum state of the electromagnetic field will then be described by the coherent state associated with the classical retarded solution.
- The expected number of horizon photons in this state is $\langle N \rangle_{\mathscr{H}^+} = ||A_\Omega||^2_{\mathscr{H}^+}$ where A_Ω is the classical retarded solution.

Quantum state of electromagnetic radiation

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2022]

Choosing a suitable gauge $A_\mu n^\mu = 0$, we have the number of photons radiated through the horizon is given by $\langle N \rangle_{\mathscr{H}^+} = \frac{1}{\pi\hbar} \int r^4 d\Sigma \int_0^\infty \omega d\omega |\hat{A}_\Omega(\omega, x^\Omega)|^2$ where $\hat{A}_\Omega$ is the Fourier transform of A_Ω with respect to the affine V.

Infinite soft photons

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2022]

- In this gauge, $E_\Omega = -\partial_V A_\Omega$, hence A_Ω does not return to its initial value at late times. Analogous to memory effect.
- Implies that $\hat{A}_\Omega$ diverges as $\frac{1}{\omega}$ as $\omega \to 0$, which implies that $\langle N \rangle_{\mathscr{H}^+} \to \infty$. This is a precise analogue of the infrared divergences in scattering theory (for $d = 4$).
- If the charge remains in its new position forever, the number of photons radiated into the black hole is infinite.

Estimating ΔA_Ω

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2022]

- Radial electric field of a point charge located a distance b from the black hole $E_r \sim \frac{q}{b^2}$. If the charge is moved a distance $d \ll b$, we have $\Delta E_r \sim \frac{qd}{b^3}$.
- Since $\Delta E = -\int dV \mathcal{D}^\Omega E_\Omega = \int dV \mathcal{D}^\Omega (\partial_V A_\Omega) = \mathcal{D}^\Omega (\Delta A_\Omega)$.
- Hence, at $r \sim M$, we have $\Delta A_\Omega \sim \frac{M^2 qd}{b^3}$. Eventually, when the particle is moved back, the change in ΔA_Ω will be equal and opposite.

Avoiding the Divergence

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2022]

- The infrared divergence is avoided if the charge is moved back to its original position.
- But if the charge is held at the point for a very long time T, the contribution will be dominated by the low-frequency contribution arising from the time interval over which $\Delta A_\Omega \sim \frac{M^2 qd}{b^3}$.

Derivation of $\langle N \rangle_{\mathscr{H}^+}$

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2022]

- For a step–like change in the angular potential, we have $\hat{A}_\Omega(\omega) \sim \frac{\Delta A_\Omega}{\omega}$.
- Thus, the frequency integral becomes $\int_{\frac{1}{V}}^{\infty} \omega d\omega \frac{|\Delta A_\Omega|^2}{\omega^2} \sim |\Delta A_\Omega|^2 \ln V$.
- Thereby, the number of photon influx $\langle N \rangle_{\mathscr{H}^+} \sim |\Delta A_\Omega|^2 \ln V \sim \frac{M^4 q^2 d^2}{b^6} \ln V$.

Estimate of $\langle N \rangle_{\mathscr{H}^+}$

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2022]

- For a black hole, the relation between affine time and Killing time is $V = \exp\left(\frac{\kappa v}{c}\right)$, where $\kappa = \frac{1}{4M}$ is the surface gravity.
- The Killing time is related to the particle's proper time by the redshift factor, which is of the order of unity near the horizon.
- The final estimate of $\langle N \rangle_{\mathscr{H}^+} \sim \frac{M^3 q^2 d^2}{b^6} T$.

Analysing Coherence

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2022]

- After passing through the SG apparatus, the first component of the particle remains at a position, and the second component of her particle moves d away.
- Recombination after time t at the same position as the first particle. No radiation was emitted by the first component, hence $|\mathscr{A}_L\rangle_{\mathscr{H}^+} = |0\rangle_{\mathscr{H}^+}$.
- In state, $|\mathscr{A}_R\rangle_{\mathscr{H}^+}$, we have expected number of photons as $\langle N\rangle_{\mathscr{H}^+} \sim \frac{M^3q^2d^2}{b^6}T$. Hence, due to the black hole decoherence happens in $T \sim \frac{1}{\frac{M^3q^2d^2}{b^6}}$ time due to $\langle N\rangle_{\mathscr{H}^+} \gtrsim 1$.

Gravitational Coherence

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2022]

- The electric components equivalent to the electromagnetic case, is formed from the Weyl tensor $C_{\mu\nu\eta\rho}$ by contraction with the four-velocity vectors given by $E_{\mu\nu} = C_{\mu\nu\eta\rho}n^{\eta}n^{\rho}$.
- For a static point mass outside a Schwarzschild black hole the only non-vanishing component of the electric part of the Weyl tensor on the horizon is $E_{rr} = C_{\mu\nu\eta\rho}\ell^{\mu}n^{\eta}\ell^{\nu}\eta^{\rho}$.

Gravitational Coherence

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2022]

- Gravitational radiation on the horizon is described by the pullback $E_{\Omega\Theta}$ of $E_{\mu\nu}$, which vanishes for a static point mass.
- The process of moving the particle quasi-statically to a new location will involve a change in E_{rr}.
- The effective Maxwell equations from the Bianchi identity gives $\mathcal{D}^{\Omega}E_{\Omega\Theta} = -\partial_V E_{r\Theta}$ and $\mathcal{D}^{\Theta}E_{r\Theta} = -\partial_V E_{rr}$, thereby $\mathcal{D}^{\Omega}\mathcal{D}^{\Theta}E_{\Omega\Theta} = \partial_V^2 E_{rr}$.

Gravitational Coherence

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2022]

- Quasi-static motion results in the emission of radiation. Linearized perturbation theory to study the number of gravitons emitted.
- For $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, in a gauge where $h_{\mu\nu}n^{\mu} = 0$ and the induced angular metric $\delta^{\Omega\Theta}h_{\Omega\Theta} = 0$ on the horizon, we have the free data to be specified as $h_{\Omega\Theta}$.
- As in the electromagnetic case, a particle in the Fock space associated with the Hartle-Hawking vacuum is a solution with positive frequency with respect to affine parameter V, with $E_{\Omega\Theta} = -\frac{1}{2}c^2\partial_V^2 h_{\Omega\Theta}$.

Is a Black Hole essential?

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2023]

- The mere presence of the black hole implies a fundamental rate of decoherence on the quantum superposition. Generalization to Killing Horizons.
- Spatial superposition that is kept stationary with respect to the symmetry generating the Killing horizon will decohere.

Killing Vector Fields

[Witten, Edward, 2024]

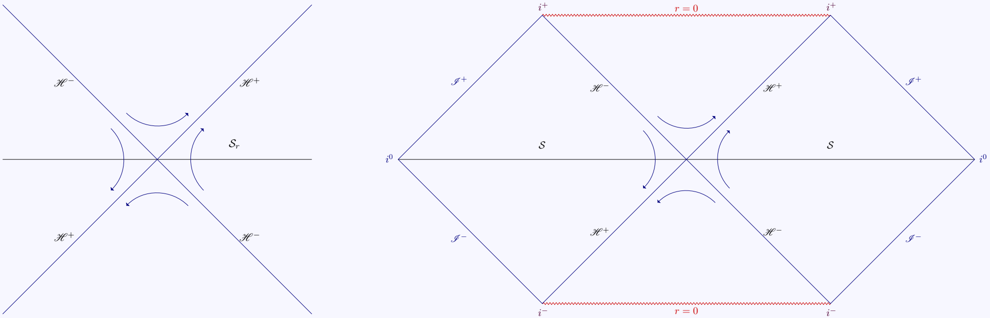

Figure: Symmetries and corresponding KVFs for Rindler and Schwarzschild spacetimes

Rindler Killing Horizons

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2023]

- Stationery under Lorentz Boosts: Uniformly Accelerating frame.
- Decoherence of a uniformly accelerating spatially separated superposition occurs because of the emission of soft gravitons.
- Decoherence independent of Unruh radiation.

Analysis of Decoherence

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2023]

- Consider a charged particle in a stationary spacetime being in a superposition of two spacelike separated states $|\psi_1\rangle$ and $\psi_2\rangle$ with $|\Psi\rangle = \alpha|\psi_1\rangle + \beta|\psi_2\rangle$.
- Fluctuations in the charge-current operator $\mathbf{j}^\mu$ of the states are negligibly small over the scales of interest. Approximating the functionals with their respective eigenvalues $j_1^\mu = \langle\psi_1|\mathbf{j}^\mu|\psi_1\rangle$ and $j_2^\mu = \langle\psi_2|\mathbf{j}^\mu|\psi_2\rangle$.
- At early times, electromagnetic field observable $\mathbf{A}_\mu^{in} = \mathbf{A}_\mu - \mathbf{C}_\mu^{in}\mathbb{I}$, assuming stationary charge with Couloumb field component $\mathbf{C}_\mu^{in}$.

Analysis of Decoherence

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2023]

- We extend $\mathbf{A}_{\mu}^{in}$ to satisfy the source-free Maxwell equations at all times. The radiation state determines the initial state of the electromagnetic field.
- If the spacetime were globally stationary (timeline Killing horizon), no Killing horizons. The initial state of the radiation is the stationary vacuum state.

Field Quantization

[Hollands, Stefan, and Robert M. Wald, 2015]

- Operator valued distribution for any smooth, compactly supported test function q, $\phi(q) = \int_{\mathcal{M}} q(x)\phi(x)d\Sigma_g$, for well-definedness of the quantization.
- We define a suitable algebra $\mathscr{A}(\mathcal{M}, g)$, of quantum observables ($\phi(x)$ is an $\mathscr{A}$ valued distribution), accounting the distributional nature of the field, the field equation, the real character of ϕ and the symplectic structure of the classical phase space of this theory.
- We have the correspondence $[\phi(q_1), \phi(q_2)] = iA(q_1, q_2)\mathbb{I}$, where $A(q_1, q_2) = A^+(q_1, q_2) - A^-(q_1, q_2)$, the anti-symmetric combination of the advanced and retarded propagators.

Field Quantization

[Hollands, Stefan, and Robert M. Wald, 2015]

- A physical state, Ψ, is an expectation value functional, a linear map $\Psi : \mathscr{A}(\mathcal{M}, g) \to \mathbb{C}$ satisfying the normalization condition $\Psi(\mathbb{I}) = 1$, and positivity, $\Psi(a^\dagger a) \geq 0 \; \forall a \in \mathscr{A}$.
- Any state is described by the collection $\{\Pi_n\}_{n\geq 1}$ of its n-point functions:

$$\Pi_n(f_1, \cdots, f_n) = \Psi\big(\phi(f_1), \ldots \phi(f_n)\big).$$

Constructing the Hilbert space

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2023]

- As on every globally hyperbolic spacetime, we quantize the field $\mathbf{A}_\mu^{in}$ with the initial vacuum state $|\Psi_0\rangle$ invariant under the time translation symmetries, by viewing it (after smearing with a test function) as an element of the associated abstract algebra $\mathscr{A}(\mathcal{M}, g)$.
- We construct a one-particle Hilbert space $\mathcal{H}_{in}$, and corresponding Fock space $\mathcal{F}_{in}$, with the $\mathbf{A}_\mu^{in}(q^\mu) = i\{\mathbf{a}(\overline{\tilde{\sigma}_q}) - \mathbf{a}^\dagger(\tilde{\sigma}_q)\}$. Here σ_q is the advanced minus retarded solution with divergence-free source q^μ (for removing gauge dependency), and $\tilde{}$ denotes representation in the Hilbert space.

Constructing the Hilbert space

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2023]

- The advanced minus retarded solution is given by

$$\sigma_{q^\mu}(x) = \int_{\mathcal{M}} \sqrt{-g}\, d^4x' \{G^{adv}_{\mu\nu}(x,x') - G^{ret}_{\mu\nu}(x,x')\} q^\nu(x'),$$

is independent of the source since subtracting the $J^\pm$ causal dependency.

- Commutation relations $[\mathbf{a}(\overline{\tilde{\sigma}_q}), \mathbf{a}^\dagger(\tilde{\sigma}_q)] = \langle \tilde{\sigma}_q | \tilde{\sigma}_{q'} \rangle \mathbb{I}$ with the Klein-Gordon inner product. Crucial correspondence.
- We have $[\mathbf{A}^{in}_\mu(q^\mu), \mathbf{A}^{\dagger in}_\mu(q'^\mu)] = 2i\langle \tilde{\sigma}_q | \tilde{\sigma}_{q'} \rangle \mathbb{I}$. Thereby, the quantization is independent on the operator definition of the field.

Correlation Function

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2023]

Correlation functions of the field $\mathbf{A}_{i,\mu}$ for $|\Psi_i\rangle$ at a general time are found by solving for Maxwell's equation, by the assumption of the minimal fluctuations in the charge current. We have,

$$\begin{aligned}\langle \mathbf{A}_{i,\mu_1}(x_1)\dots\mathbf{A}_{i,\mu_n}(x_n)\rangle \\ = \langle\Psi_0|\left[\mathbf{A}^{in}_{\mu_1}(x_1) + G^{ret}_{\mu_1\nu}(j^\nu_i)(x_1)\mathbb{I}\right] \\ \dots\left[\mathbf{A}^{in}_{\mu_n}(x_n) + G^{ret}_{\mu_n\nu}(j^\nu_i)(x_n)\mathbb{I}\right]|\Psi_0\rangle.\end{aligned}$$

Late times Correlation

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2023]

- At late times, $\mathbf{A}^{out}_{\mu} = \mathbf{A}_{\mu} - \mathbf{C}^{out}_{\mu}\mathbb{I}$.
- Thereby, correlation functions at late times, $\langle \mathbf{A}^{out}_{i,\mu_1}(x_1)\dots\mathbf{A}^{out}_{i,\mu_n}(x_n)\rangle$

$$= \langle\Psi_0| \left[\mathbf{A}^{in}_{\mu_1}(x_1) + \underbrace{\{G^{ret}_{\mu_1\nu}(j^{\nu}_i)(x_1) - \mathbf{C}^{out}_{\mu_1}\mathbb{I}\}}_{\mathcal{A}_{i,\mu_1}}\right]$$

$$\dots\left[\mathbf{A}^{in}_{\mu_n}(x_n) + \underbrace{\{G^{ret}_{\mu_n\nu}(j^{\nu}_i)(x_1) - \mathbf{C}^{out}_{\mu_n}\mathbb{I}\}}_{\mathcal{A}_{i,\mu_n}}\right]|\Psi_0\rangle.$$

Coherent states

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2023, Hollands, Stefan, and Robert M. Wald, 2015]

- Displacement operator $D(\mathcal{A}_{\mu,i}) = \exp\left(\mathbf{a}(\tilde{\mathcal{A}}_{\mu,i}) - \mathbf{a}^\dagger(\tilde{\mathcal{A}}_{\mu,i})\right)$ such that $D(\mathcal{A}_{\mu,i})\mathbf{A}^{in}_\mu D^\dagger(\mathcal{A}_{\mu,i}) = \mathbf{A}^{in}_\mu + \mathcal{A}_{\mu,i}$.
- Correlation functions on any late time Cauchy surface are of the coherent state

$$|\Psi_{\mu,i}\rangle \sim \exp\left(-\frac{1}{2}||\tilde{\mathcal{A}}_{\mu,i}||^2\right)\exp\left(\mathbf{a}^\dagger(\tilde{\mathcal{A}}_{\mu,i})\right)|\Psi_0\rangle.$$

- The n-point functions of Gaussian states can be expressed entirely in terms of their 1- and 2-point functions. Any Gaussian state, Ψ, can be expressed as the vacuum state in a Fock representation of some algebra $\mathscr{A}$.

Evaluating Decoherence

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2023]

For the particle radiation system, $|\Xi_\mu\rangle \sim \alpha(|\psi_1\rangle \otimes |\Psi_{\mu,1}\rangle) + \beta(|\psi_2\rangle \otimes |\Psi_{\mu,2}\rangle)$, thereby,

$$\mathscr{D} = 1 - \underbrace{(\alpha\overline{\beta} + \overline{\alpha}\beta)}_{\equiv 1 \text{ for } \alpha=\beta=\frac{1}{\sqrt{2}}} \langle\Psi_{\mu,1}|\Psi_{\mu,2}\rangle$$

$$= 1 - \exp\left(-\frac{1}{2}(||\tilde{\mathcal{A}}_{\mu,1}||^2 + ||\tilde{\mathcal{A}}_{\mu,2}||^2)\right)\langle\Psi_0|\exp\left(\mathbf{a}(\tilde{\mathcal{A}}_{\mu,1})\right)\exp\left(\mathbf{a}^\dagger(\tilde{\mathcal{A}}_{\mu,2})\right)|\Psi_0\rangle$$

$$= 1 - \exp\left(-\frac{1}{2}(||\tilde{\mathcal{A}}_{\mu,1}||^2 + ||\tilde{\mathcal{A}}_{\mu,2}||^2)\right)\langle\Psi_0|\exp\Big(\mathbf{a}(\tilde{\mathcal{A}}_{\mu,1}) + \mathbf{a}^\dagger(\tilde{\mathcal{A}}_{\mu,2}) +$$

$$\left[\mathbf{a}(\tilde{\mathcal{A}}_{\mu,1}), \mathbf{a}^\dagger(\tilde{\mathcal{A}}_{\mu,2})\right] + \dots\Big)|\Psi_0\rangle$$

Evaluating Decoherence

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2023]

$$\begin{aligned}\textit{(continued)} &= 1 - \exp\left(-\frac{1}{2}(||\tilde{A}_{\mu,1}||^2 + ||\tilde{A}_{\mu,2}||^2)\right)\langle\Psi_0|\exp\left(\langle\mathcal{A}_{\mu,1}|\mathcal{A}_{\mu,2}\rangle\right)|\Psi_0\rangle \\ &= 1 - \exp\left(-\frac{1}{2}(||\tilde{A}_{\mu,1} - \tilde{A}_{\mu,2}||^2)\right).\end{aligned}$$

- For late time states $\tilde{A}_{\mu,1} - \tilde{A}_{\mu,2} = G^{ret}_{\mu\nu}(j_1^\nu)(x) - G^{ret}_{\mu\nu}(j_2^\nu)(x) = G^{ret}_{\mu\nu}(j_1^\nu - j_2^\nu)(x)$.
- Entangling photons as the coherent state $j_1^\nu - j_2^\nu$, with $\langle N\rangle \sim ||\tilde{G}^{ret}_{\mu\nu}(j_1^\nu - j_2^\nu)||^2$, hence $\mathscr{D} = 1 - \exp\left(-\frac{1}{2}(||\tilde{G}^{ret}_{\mu\nu}(j_1^\nu - j_2^\nu)||^2)\right) = 1 - \exp(-\frac{1}{2}\langle N\rangle)$.

Rindler Horizons

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2023]

- Uniformily accelerating A with $x_A^\mu \equiv (\frac{1}{a}\sinh(a\tau), 0, 0, \frac{1}{a}\cosh(a\tau))$ in the orbit of the boost $b^\mu \equiv (az, 0, 0, at)$ with normalization $b^\mu b_\mu = -1$.
- In null coordinates $U, V \equiv t \mp z$ with vector fields $n^\mu = \partial_V^\mu$ and $\ell^\mu = \partial_U^\mu$ which are future-directed null vectors with $\ell^\mu n_\nu = -1$.
- The boost Killing field $b^\mu = a(-U\ell^\mu + Vn^\mu)$ is null on the two Rindler horizons $U = 0 \ (\mathscr{H}_R^+), \ V = 0 \ (\mathscr{H}_R^-)$.

Soft Photons Decoherence

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2023, Unruh, William G., and Robert M. Wald, 1984]

- Initial vacuum state of the field. A stationary with respect to the Killing vector field. Quantum superposition for time T and recombination.
- We consider the future Rindler Horizon $\mathscr{H}_R^+$ as an *effective* Cauchy surface, assuming solutions that are stable at late times, for the wave equation.
- The out states $|\Psi_{\mu,1}\rangle$ and $|\Psi_{\mu,2}\rangle$ of the radiation are completely determined by data on $\mathscr{H}_R^+$. Note that this contrasts with the black hole case, where one would need data on $\mathscr{H}^+$ and $\mathscr{I}^+$.

Electrodynamics in Rindler spacetime

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2023]

- For classical stationary point charge q accelerating a in the right Rindler wedge $\mathcal{W}_r$ and lies at proper distance D, such that $D = \frac{1}{a}$, from the bifurcation surface of the Rindler horizon.
- Non-vanishing radiation component is $E_U = F_{\mu\nu}\ell^\mu n^\nu = \frac{2qa^2}{\pi(1+a^2\rho^2)^2}$, and the dual field tensor describes the radiation through the horizon by $E_\mu = F_{\mu\nu}n^\mu$ to $\mathscr{H}_R^+$.

Electrodynamics in Rindler spacetime

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2023]

- Since $E_\Omega = 0$ on the horizon for a uniformly accelerated charge, it does not produce any radiation as determined on $\mathscr{H}_R^+$ even though a uniformly accelerated charge radiates energy to $\mathscr{I}^+$.
- We study the case of a change in the orbit of an accelerated observer through Maxwell's equation $\mathcal{D}^\Omega E_\Omega = \partial_V E_U$. As earlier, in the transverse gauge, we have $E_\Omega = -\partial_V A_\Omega$.
- Since the transverse components of the Coulomb field of a static charge vanish, we may replace $E_\Omega = -\partial_V \mathcal{A}_\Omega$.

Memory Effect of the Potential

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2023]

- From the Maxwell's equation $\Delta E_U = -\mathcal{D}^{\Omega}(\Delta \mathcal{A}_{\Omega})$, where $E_U \sim \frac{-4qda^3(1-a^2\rho^2)}{\pi(1+a^2\rho^2)^3}$.
- Even though $E_{\Omega} = 0$ at both late and early times, $\mathcal{A}_{\Omega}$ does not return to its original value at late times.
- Considering the quantized radiation through the horizon, for the electromagnetic field on $\mathscr{H}_R^+$, the *free data* on $\mathscr{H}_R^+$ is the pullback of the vector potential.

Infrared Divergences

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2023]

The expected number of photons on $\mathscr{H}^+$ in the coherent state associated with any classical solution $\mathcal{A}_\Omega$ is,

$$\langle N \rangle_{\mathscr{H}_R^+} = ||\tilde{\mathcal{A}}||^2_{\mathscr{H}_R^+} = 2 \int_{\mathbb{R}^2} d\Omega \int_0^\infty \frac{\omega d\omega}{2\pi} |\hat{\mathcal{A}}_\Omega(\omega, x^\Omega)|^2,$$

where $\hat{\mathcal{A}}_\Omega$ is the Fourier transform of $\mathcal{A}_\Omega$ with respect to the affine parameter V.

Gravitational Analogue

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2022]

- The electric components equivalent to the electromagnetic case, is formed from the Weyl tensor $C_{\mu\nu\eta\rho}$ by contraction with the four-velocity vectors given by $E_{\mu\nu} = C_{\mu\nu\eta\rho}n^{\eta}n^{\rho}$.
- The effective Maxwell equations from the Bianchi identity gives $\mathcal{D}^{\Omega}E_{\Omega\Theta} = -\partial_V E_{U\Theta}$ and $\mathcal{D}^{\Theta}E_{r\Theta} = -\partial_V E_{UU}$, thereby $\mathcal{D}^{\Omega}\mathcal{D}^{\Theta}E_{\Omega\Theta} = \partial_V^2 E_{UU}$.

Gravitational Analogue

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2022]

- Quasi-static motion results in the emission of radiation. Linearized perturbation theory to study the number of gravitons emitted.
- For $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, in a gauge where $h_{\mu\nu}n^{\mu} = 0$ and the induced angular metric $\delta^{\Omega\Theta}h_{\Omega\Theta} = 0$ on the horizon, we have the free data to be specified as $h_{\Omega\Theta}$.
- As in the electromagnetic case, a particle in the Fock space associated with the Poincare-invariant vacuum is a solution with positive frequency with respect to affine parameter V, with $E_{\Omega\Theta} = -\frac{1}{2}\partial_V^2 h_{\Omega\Theta}$.

Unruh Effect

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2022, Witten, Edward, 2024]

- Thermalisation of an accelerated observer with temperature $\mathscr{T} \sim \frac{a}{2\pi}$, relative to the notion of time translations by the boost.
- Collisional decoherence due to scattering of radiation. Assuming elastic scattering with $\lambda \gg d$.
- When an Unruh photon elastically scatters off the particle, the outgoing state will depend on which branch it interacts with. For $|\xi_i\rangle$ photon state from $|\psi_i\rangle$, with an estimate $\mathscr{D} = 1 - \langle \xi_1 | \xi_2 \rangle$.

Scattering Amplitudes

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2022]

- In the Rindler frame, the particle is immersed in a bath of Unruh photons with a thermal momentum distribution $\rho(p) \sim \frac{p^2}{\exp(\frac{p}{\mathscr{T}})-1}$[2].
- For the outgoing states, we have $|\chi_2\rangle \approx e^{-i\vec{p}\cdot\vec{d}}|\chi_1\rangle$. Expanding $e^{-i\vec{p}\cdot\vec{d}}$, implying a single-event decoherence factor $\mathscr{D} \sim (pd)^2$.
- To obtain the total decoherence rate, we integrate over the thermal photon spectrum. The differential decoherence rate is $d\Gamma_{\text{scatt}} \sim (d^2p^2)\rho(p)\sigma(p)dp$, where the scattering cross-section in the Thomson limit is $\sigma_T = \frac{8\pi}{3}\left(\frac{q^2}{4\pi m}\right)^2$.

[2] Note, here we are assuming incoherent scattering effects of individual Unruh photons.

Decoherence from Unruh Photon Scattering

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2022]

- Thus, for scattering $\Gamma_{\text{scatt}} \sim d^2\sigma_T \int_0^\infty \frac{p^4 dp}{\exp(\frac{p}{\mathscr{T}}) - 1} \sim d^2\sigma_T \mathscr{T}^5 \sim \frac{q^4 d^2 a^5}{m^2}$.
- For radiation of soft Rindler horizon photons, $\mathscr{D} \approx 1 - \exp(-\Gamma_{\text{rad}}\mathscr{T}) \approx \Gamma_{\text{rad}}\mathscr{T}$, for $\Gamma_{\text{rad}}\mathscr{T} \ll 1$, where decoherence rate is $\Gamma_{\text{rad}} = q^2 d^2 a^3$.
- The relative ratio is bound by the charge radius $\frac{q}{m}$, represents a fundamental lower bound to the spread of a charged particle due to vacuum fluctuations of the electromagnetic field, hence $\Gamma_{\text{scatt}} \ll \Gamma_{\text{rad}}$.

Is a Horizon essential?

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

Is the global properties of spacetime, specifically the presence of a horizon, essential for the description of the decoherence phenomenon? Local description through the properties of the unperturbed quantum field.

Decoherence Computation

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- $\mathbf{A}^{in}_{\mu}(q^{\mu}) = i\{\mathbf{a}(\overline{\tilde{\sigma}_q}) - \mathbf{a}^{\dagger}(\tilde{\sigma}_q)\}$, hence,

$$\begin{aligned}\langle\Psi_0|\left(\mathbf{A}^{in}_{\mu}(q^{\mu})\right)^2|\Psi_0\rangle &= \langle\Psi_0|\left[\mathbf{a}(\overline{\tilde{\sigma}_q}), \mathbf{a}^{\dagger}(\tilde{\sigma}_q)\right]|\Psi_0\rangle \\ &= \langle\tilde{\sigma}_q|\tilde{\sigma}_q\rangle = ||\tilde{\sigma}_q||^2\end{aligned}$$

- We relate it to the number of entangling photons $\langle N\rangle \sim ||\tilde{G}^{ret}_{\mu\nu}(j_1^{\nu} - j_2^{\nu})||^2$ by $\sigma_{q^{\mu}}(x) = \int_{\mathcal{M}} \sqrt{-g}\; d^4x'\{G^{adv}_{\mu\nu}(x,x') - G^{ret}_{\mu\nu}(x,x')\}q^{\nu}(x')$.
- Note that $\sigma_{q^{\mu}}$ is valid at all times, while $\langle N\rangle$ is valid only at late times accounting to $j_1^{\nu} - j_2^{\nu} \to 0$

Green's Function

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- At late times, the currents become identical, for a source that is nonzero only in a finite time interval, outside the region, the advanced solution vanishes, hence $G^{ret}_{\mu\nu}(j^\nu_1 - j^\nu_2) = -\left\{G^{adv}_{\mu\nu}(j^\nu_1 - j^\nu_2) - G^{ret}_{\mu\nu}(j^\nu_1 - j^\nu_2)\right\}$.
- Further note that in our context the source is the difference of the currents, $q(x) = j_1(x) - j_2(x)$, thereby, the classical field

$$\sigma_{j^\nu_1 - j^\nu_2}(x) = \int_{\mathcal{M}} \sqrt{-g}\ d^4x' \{G^{adv}_{\mu\nu}(j^\nu_1 - j^\nu_2)(x,x') - G^{ret}_{\mu\nu}(j^\nu_1 - j^\nu_2)(x,x')\}$$
$$= -\int_{\mathcal{M}} \sqrt{-g}\ d^4x' G^{ret}_{\mu\nu}(j^\nu_1 - j^\nu_2)(x,x').$$

Decoherence Computation

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

Hence,

$$\begin{aligned}\langle\Psi_0|\left(\mathbf{A}^{in}_{\mu}(j_1^{\mu}-j_2^{\mu})\right)^2|\Psi_0\rangle &= ||\tilde{\sigma}_{j_1^{\nu}-j_2^{\nu}}||^2 \\ &= \left|-\int_{\mathcal{M}}\sqrt{-g}\; d^4x'\tilde{G}^{ret}_{\mu\nu}(j_1^{\nu}-j_2^{\nu})(x,x')\right|^2 \\ &\sim ||\tilde{G}^{ret}_{\mu\nu}(j_1^{\nu}-j_2^{\nu})||^2 = \langle N\rangle.\end{aligned}$$

Prescription

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- We compute the expected currents j_1^ν, j_2^ν of the components of the superposition.
- We compute the two-point function $\langle \Psi_0 | \mathbf{A}_\mu^{in}(x) \mathbf{A}_\mu^{in}(x') | \Psi_0 \rangle$ of the unperturbed field in the vacuum state.
- We smear this two-point function in both variables with the divergence-free test vector field $q^\nu = j_1^\nu - j_2^\nu$.
- We have $\mathscr{D} = 1 - \exp(-\frac{1}{2}\langle N \rangle) = 1 - \exp\left[-\frac{1}{2} \langle \Psi_0 | \left(\mathbf{A}_\mu^{in}(j_1^\mu - j_2^\mu) \right)^2 | \Psi_0 \rangle \right]$.

Decoherence in the Unruh Vaccum of a Black Hole

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- Consider the hypersurface Σ_t orthogonal to timelike Killing vector field t^μ. For the path $\underset{\sim}{X}_k(t)$ taken by the k^{th} component, we have the current densities on the surface $j_k^\mu(t, \underset{\sim}{x}) \approx \frac{q}{\sqrt{-g}} \delta^{(3)}\left(\underset{\sim}{x} - \underset{\sim}{X}_k(t)\right) u_k^\mu \frac{d\tau_k}{dt}$.
- For non-relativistic motion relative to t^μ, we approximate $d\tau_k^2 \approx -g_{tt} dt^2$ $\frac{d\tau_k}{dt} \approx \sqrt{-g_{tt}}$, such that we decompose $u_k^\mu = \left(\frac{dX^t}{d\tau}, \frac{dX_k^\mu}{d\tau}\right) = \frac{t^\mu}{\sqrt{-g_{tt}}} + \frac{dX_k^\mu}{d\tau}$
- Thereby, $j_k^\mu(t, \underset{\sim}{x}) \approx \frac{q}{\sqrt{-g}} \delta^{(3)}\left(\underset{\sim}{x} - \underset{\sim}{X}_k(t)\right) \left(t^\mu + \frac{dX_k^\mu}{dt}\right)$ and $\frac{dX^t}{dt} = 0$.

Displacement Analysis

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- Suppose the displacement of the two components is given by $S^\mu(t) = d(t)s^\mu$, where s^μ is the deviation vector between the curves.
- We impose the proper distance as $d(t) = \begin{cases} 0 & t < -\frac{T}{2} - T_1 \\ d & -\frac{T}{2} < t < \frac{T}{2} \\ 0 & t > \frac{T}{2} + T_2 \end{cases}$.
- We have the relative positions with respect to the lab as $\underset{\sim}{X}_1(t) = \underset{\sim}{X}(t) - \frac{1}{2}d(t)\underset{\sim}{s}$, and $\underset{\sim}{X}_2(t) = \underset{\sim}{X}(t) + \frac{1}{2}d(t)\underset{\sim}{s}$.

Electrodynamics of the superposition

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- $\delta^{(3)}\left(\underset{\sim}{x} - \underset{\sim}{X}(t) \pm \frac{1}{2}d(t)\underset{\sim}{s}\right) \sim \delta^{(3)}(\underset{\sim}{x} - \underset{\sim}{X}(t)) \mp \frac{1}{2}s^\nu\nabla_\nu\left[d(t)\delta^{(3)}(\underset{\sim}{x} - \underset{\sim}{X}(t))\right].$ Identity based on the smearing of a continuous function.
- The difference in current densities is

$$\begin{aligned}
j_1^\mu - j_2^\mu \approx{}& \frac{q}{\sqrt{-g}}\delta^{(3)}\left(\underset{\sim}{x} - \underset{\sim}{X}(t) - \frac{1}{2}d(t)\underset{\sim}{s}\right)\left(t^\mu + \frac{1}{2}\frac{d\ d(t)}{dt}s^\mu\right) \\
&- \frac{q}{\sqrt{-g}}\delta^{(3)}\left(\underset{\sim}{x} - \underset{\sim}{X}(t) + \frac{1}{2}d(t)\underset{\sim}{s}\right)\left(t^\mu - \frac{1}{2}\frac{d\ d(t)}{dt}s^\mu\right) \\
\approx{}& \frac{q}{\sqrt{-g}}s^\nu\nabla_\nu\left[d(t)\delta^{(3)}(\underset{\sim}{x} - \underset{\sim}{X}(t))\right]t^\mu - \frac{q}{\sqrt{-g}}\delta^{(3)}(\underset{\sim}{x} - \underset{\sim}{X}(t))s^\mu t^\nu\nabla_\nu d(t).
\end{aligned}$$

Electric field of the superposition

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- Hence, $j_1^\mu - j_2^\mu \approx \frac{2q}{\sqrt{-g}} t^{[\mu} s^{\nu]} \nabla_\nu \left[d(t) \delta^{(3)} \left(\underset{\sim}{x} - \underset{\sim}{X}(t) \right) \right]$.
- We have electric field on the surface as $E_\mu = F_{\mu\nu} t^\nu = \nabla_{[\mu} A^{\nu]} t^\nu$. Thereby,

$$
\begin{aligned}
\mathbf{A}_\mu^{in}(j_1^\mu - j_2^\mu) &= \int d^4x \sqrt{-g} A_\mu^{in}(x)(j_1^\mu - j_2^\mu)(x) \\
&\approx 2q \int d^4x \; t^\mu s^\nu A_{[\mu}^{in}(x) \nabla_{\nu]} \left[d(t) \delta^{(3)} \left(\underset{\sim}{x} - \underset{\sim}{X}(t) \right) \right] \\
&\approx 2q \int dt \; d(t) \int d^3x \; t^\mu s^\nu A_{[\mu}^{in}(x) \nabla_{\nu]} \delta^{(3)} \left(\underset{\sim}{x} - \underset{\sim}{X}(t) \right).
\end{aligned}
$$

Decoherence of the superposition

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- Hence, we have

$$\begin{aligned}\mathbf{A}^{in}_{\mu}(j_1^{\mu}-j_2^{\mu}) &\approx 2q\int dt\ d(t)\int d^3x\ t_{\nu}s^{\mu}\nabla_{[\mu}A^{\nu]}(x)\delta^{(3)}\left(\underset{\sim}{x}-\underset{\sim}{X}(t)\right)\\ &\approx 2q\int dt\ d(t)s^{\mu}\mathbf{E}^{in}_{\mu}(t,\underset{\sim}{X}(t))\end{aligned}$$

- Thereby, $\mathscr{D}=1-\exp(-\frac{1}{2}\langle N\rangle)$ where $\langle N\rangle=\langle\Psi_0|\Big(\mathbf{A}^{in}_{\mu}(j_1^{\mu}-j_2^{\mu})\Big)^2|\Psi_0\rangle\approx$ $4q^2\int dt\ dt'\ d(t)\ d(t')\langle\Psi_0|s^{\mu}\mathbf{E}^{in}_{\mu}(t,\underset{\sim}{X}(t))s^{\nu}\mathbf{E}^{in}_{\nu}(t',\underset{\sim}{X}(t'))|\Psi_0\rangle$

Electric Field Two-Point Function

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- The two-point function of the component $s^\mu \mathbf{E}^{in}_\mu$ of the electric field in the direction of separation, at the lab coordinates, corresponds to the entangling photons.
- We evaluate the two-point function by a mode expansion. For simplicity, assuming radial separation, such that the radial electric field $\mathbf{s}^r \mathbf{E}_r(x) = \partial_0 \mathbf{A}_r - \partial_r \mathbf{A}_0$, hence we compute $\langle \Psi_0 | \mathbf{E}_r(x) \mathbf{E}_r(x') | \Psi_0 \rangle = \langle \Psi_0 | \big(\partial_0 \mathbf{A}_r - \partial_r \mathbf{A}_0 \big) \big(\partial'_0 \mathbf{A}_r - \partial'_r \mathbf{A}_0 \big) | \Psi_0 \rangle$.

Electric Field Two-Point Function

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

$$\langle\Psi_0|\mathbf{E}_r(x)\mathbf{E}_r(x')|\Psi_0\rangle = \sum_{\ell=1}^{\infty}\frac{1}{16\pi^2}\ell(\ell+1)(2\ell+1)\frac{P_\ell(\hat{r},\hat{r}')}{r^2r'^2}\int_{-\infty}^{\infty}\frac{d\omega}{\omega}\exp(-i\omega(t-t'))$$
$$\times\left[G^+(\omega)R^+_{\omega\ell}(r)\overline{R^+_{\omega\ell}(r')}+G^-(\omega)R^-_{\omega\ell}(r)\overline{R^-_{\omega\ell}(r')}\right],$$

where the mode functions satisfy $\frac{d^2R_{\omega\ell}}{dr^{*2}}+\left\{\omega^2-\left(1-\frac{2M}{r}\right)\frac{\ell(\ell+1)}{r^2}\right\}R_{\omega\ell}=0$, with $r^*=r+2M\ln\left(\frac{r}{2M}-1\right)$ with $R^\pm_{\omega\ell}$ defined by waves that are incoming from the white hole $(+)$ and waves that are incoming from infinity $(-)$, weighted by $G^\pm(\omega)$ dependent on choice of vacuum.

Effective Entangling Photons

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

$$
\begin{aligned}
\langle N\rangle &= 4q^2\int dt\ dt'\ d(t)\ d(t')\langle\Psi_0|s^\mu \mathbf{E}^{in}_\mu(t,\underset{\sim}{X}(t))s^\nu \mathbf{E}^{in}_\nu(t',\underset{\sim}{X}(t'))|\Psi_0\rangle \\
&= 4q^2\int dt\ dt'\ d(t)\ d(t')\sum_{\ell=1}^{\infty}\frac{C_\ell s^r s_r P_\ell(\hat{r},\hat{r}')}{r^2 r'^2}\bigg|_{\hat{r}=\hat{r}'}\int_{-\infty}^{\infty}\frac{d\omega}{\omega}\exp(-i\omega(t-t')) \\
&\quad\times\left[G^+(\omega)R^+_{\omega\ell}(r)\overline{R^+_{\omega\ell}(r')}\Big|_{\hat{r}=\hat{r}'}+G^-(\omega)R^-_{\omega\ell}(r)\overline{R^-_{\omega\ell}(r')}\Big|_{\hat{r}=\hat{r}'}\right] \\
&= 4q^2\int_{-\infty}^{\infty}\frac{d\omega}{\omega}\left|\hat{d}(\omega)\right|^2\sum_{\ell=1}^{\infty}\frac{C_\ell\left(1-\frac{2M}{r}\right)}{r^4}\left[G^+(\omega)\left|R^+_{\omega\ell}(r)\right|^2+G^-(\omega)\left|R^-_{\omega\ell}(r)\right|^2\right]
\end{aligned}
$$

Effective Entangling Photons

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- Estimating the contribution to the integral,

$$d(t) = \begin{cases} 0 & t < -\frac{T}{2} - T_1 \\ d & -\frac{T}{2} < t < \frac{T}{2} \\ 0 & t > \frac{T}{2} + T_2 \end{cases} \implies \left|\hat{d}(\omega)\right| = \begin{cases} 0 & |\omega| < \frac{1}{T} \\ \frac{d}{|\omega|} & \frac{1}{T} < |\omega| < \frac{1}{\min(T_1,T_2)} \\ 0 & |\omega| > \frac{1}{\min(T_1,T_2)} \end{cases}.$$

- At large T, near the low-frequency end $|\omega| \sim \frac{1}{T}$, we determine the behaviour of the mode functions $R^{\pm}_{\omega\ell}$ by fitting the solutions in the regions described by turning points.

Turning points in the Potential

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- $$\frac{d^2R_{\omega\ell}}{dr^{*2}} + \Big\{\omega^2 - \underbrace{\left(1-\frac{2M}{r}\right)\frac{\ell(\ell+1)}{r^2}}_{V(r)}\Big\}R_{\omega\ell} = 0$$
- $r_1 = 2M + \frac{8\omega^2M^3}{\ell(\ell+1)}$ and $r_2 = \frac{\sqrt{\ell(\ell+1)}}{\omega}$.
- Regions $\begin{cases} \text{I.} & 2M < r \le r_1 \\ \text{II.} & r_1 \le r \le r_2 \\ \text{III.} & 3M \le r < \infty \end{cases}$

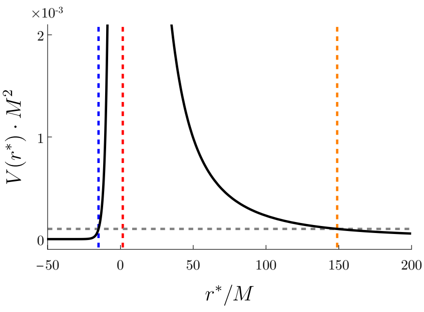

Figure: Potential $V(r^*)$ for $\ell = 1$

Potential Analysis: Regions I-III

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025, Fabbri, R., 1975]

- For Region I, $\omega^2 \gg V(r)$, hence $\frac{d^2 R_{\omega\ell}}{dr^{*2}} + \omega^2 R_{\omega\ell} = V(r) R_{\omega\ell} \approx 0$, $R^{\mathrm{I}}_{\omega\ell} \approx \alpha^{\mathrm{I}}_\ell(\omega)\exp(i\omega r^*) + \beta^{\mathrm{I}}_\ell(\omega)\exp(-i\omega r^*)$.
- For Region II, $V(r) \gg \omega^2$, hence $\frac{d^2 R_{\omega\ell}}{dr^{*2}} - V(r) R_{\omega\ell} = \omega^2 R_{\omega\ell} \approx 0$, hence we have

$$
\begin{aligned}
R^{\mathrm{II}}_{\omega\ell} \approx \alpha^{\mathrm{II}}_\ell(\omega)&\left[\frac{r}{2M} P_\ell\,(r/M-1) - \frac{1}{2(2\ell+1)}\left\{P_{\ell+1}(r/M-1) - P_{\ell-1}(r/M-1)\right\}\right] \\
&+ \beta^{\mathrm{II}}_\ell(\omega)\left[\frac{r}{2M} Q_\ell\,(r/M-1) - \frac{1}{2(2\ell+1)}\left\{Q_{\ell+1}(r/M-1) - Q_{\ell-1}(r/M-1)\right\}\right]
\end{aligned}
$$

by solving the hypergeometric equation, where P_ℓ is the Legendre polynomial and Q_ℓ is the Legendre function of the second kind.

Potential Analysis: Regions I-III

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025, Fabbri, R., 1975]

- For Region III, $\frac{d^2 R_{\omega\ell}}{dr^{*2}} + \omega^2 R_{\omega\ell} = V(r) R_{\omega\ell} \approx \frac{\ell(\ell+1)}{r^{*2}} R_{\omega\ell}$, hence approximating flat spacetime results as $R^{\text{III}}_{\omega\ell} \approx \alpha^{\text{III}}_{\ell}(\omega) r^* j_\ell(\omega r^*) + \beta^{\text{III}}_{\ell}(\omega) r^* n_\ell(\omega r^*)$ where j_ℓ and n_ℓ denote the spherical Bessel and Neumann functions.
- For $\omega M \ll 1$, we have a large overlap between Regions II and III. In this regime, we might replace r^* by r, which can cause an arbitrarily large phase error in the solutions, but at large $r \to \infty$, hence we have

$$R^{\text{II,III}}_{\omega\ell} \xrightarrow{\omega r^* \ll 1} \alpha^{\text{III}}_{\ell}(\omega) r^* \left(\frac{(\omega r^*)^\ell}{(2\ell+1)!!} \right) + \beta^{\text{III}}_{\ell}(\omega) r^* \left(\frac{-(2\ell-1)!!}{(\omega r^*)^{\ell+1}} \right)$$

$$\approx \alpha^{\text{II,III}}_{\ell}(\omega) r^{\ell+1} + \frac{\beta^{\text{II,III}}_{\ell}(\omega)}{r^\ell}.$$

Determining the mode functions

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- From the asymptotic relations of the mode function, we correspond the waves that are incoming from the white hole as $R^{+}_{\omega\ell} \to B^{+}_{\omega\ell}\exp(i\omega r^{*})$ in region III. We further assume the lab is in $M \ll r \ll \frac{1}{\omega}$, hence we have significant overlap in Regions II and III.
- Matching this to the solution in Region II, since we have $\omega r \ll 1$, we use the following $\alpha^{\mathrm{II,III}}_{\ell}(\omega) r^{\ell+1}\Big|_{3M \le r \le r_2} + \frac{\beta^{\mathrm{II,III}}_{\ell}(\omega)}{r^{\ell}}\Big|_{3M \le r \le r_2} \equiv B^{+}_{\omega\ell}\exp(i\omega r^{*})\Big|_{3M \le r \le r_2}$.

Determining the mode functions

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- At the endpoints, we have $\alpha_\ell^{\text{II,III}}(\omega)(3M)^{\ell+1} + \frac{\beta_\ell^{\text{II,III}}(\omega)}{(3M)^\ell} \approx B^+_{\omega\ell}\exp(i\omega(3M))$ and $\alpha_\ell^{\text{II,III}}(\omega)r_2^{\ell+1} + \frac{\beta_\ell^{\text{II,III}}(\omega)}{r_2^\ell} \approx B^+_{\omega\ell}\exp(i\omega r_2)$, and we solve for $\alpha_\ell^{\text{II,II}}(\omega)$ and $\beta_\ell^{\text{II,II}}(\omega)$.
- Solving, we have $\alpha_\ell^{\text{II,III}}(\omega) = B^+_{\omega\ell}\frac{r_2^\ell\exp(i\omega r_2) - (3M)^\ell\exp(i\omega(3M))}{r_2^{2\ell+1} - (3M)^{2\ell+1}}$ and $\beta_\ell^{\text{II,III}}(\omega) = (3M)^\ell r_2^\ell B^+_{\omega\ell}\frac{(3M)^\ell\exp(i\omega r_2) - r_2^\ell\exp(i\omega(3M))}{r_2^{2\ell+1} - (3M)^{2\ell+1}}$.

Determining the mode functions

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- Given the solutions $\alpha_\ell^{\mathrm{II,II}}(\omega)$ and $\beta_\ell^{\mathrm{II,II}}(\omega)$, we now match the conditions at the boundary at r_1 as $\alpha_\ell^{\mathrm{I}}(\omega)\exp(i\omega r^*)\Big|_{r=r_1} + \beta_\ell^{\mathrm{I}}(\omega)\exp(-i\omega r^*)\Big|_{r=r_1} \equiv \alpha_\ell^{\mathrm{II,III}}(\omega) r^{\ell+1}\Big|_{r=r_1} + \frac{\beta_\ell^{\mathrm{II,III}}(\omega)}{r^\ell}\Big|_{r=r_1}$.
- We have $\alpha_\ell^{\mathrm{I}}(\omega)\exp(i\omega r_1) + \beta_\ell^{\mathrm{I}}(\omega)\exp(-i\omega r_1) = \alpha_\ell^{\mathrm{II,III}}(\omega) r_1^{\ell+1} + \frac{\beta_\ell^{\mathrm{II,III}}(\omega)}{r_1^\ell}$ and the corresponding derivatives $i\omega\left[\alpha_\ell^{\mathrm{I}}(\omega)\exp(i\omega r_1) - \beta_\ell^{\mathrm{I}}(\omega)\exp(-i\omega r_1)\right] = (\ell+1)\alpha_\ell^{\mathrm{II,III}}(\omega) r_1^\ell - \ell\frac{\beta_\ell^{\mathrm{II,III}}(\omega)}{r_1^{\ell+1}}$.

Determining the mode functions

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- Solving, we have $\alpha_\ell^{\mathrm{I}}(\omega) = \frac{1}{2r^{\ell+1} i\omega \exp(i\omega r_1)(i\omega r_1+1)}\left[\alpha_\ell^{\mathrm{II,III}}(\omega) r_1^{2\ell+1}((\ell+1)(i\omega r_1 + 1) + i\omega r_1) - i\omega \beta_\ell^{\mathrm{II,III}}(\omega) r_1(\ell-1) - \omega^2 r_1^2 (\alpha_\ell^{\mathrm{II,III}}(\omega) r_1^{2\ell+1} + \beta_\ell^{\mathrm{II,III}}(\omega)) - \beta_\ell^{\mathrm{II,III}}(\omega)\ell\right]$ and $\beta_\ell^{\mathrm{I}}(\omega) = \frac{\exp(i\omega r_1)}{2r^{\ell+1} i\omega}\left[\alpha_\ell^{\mathrm{II,III}}(\omega) r_1^{2\ell+1}(i\omega r_1 - \ell - 1) + (i\omega r_1 + \ell)\beta_\ell^{\mathrm{II,III}}(\omega)\right]$.
- From the relations of the mode function, close to the horizon, we have $R^+_{\omega\ell} \to \exp(i\omega r^*) + A^+_{\omega\ell}\exp(-i\omega r^*)$, which we readjust the coefficient accordingly. Thereby, we relatively set $\alpha_\ell^{\mathrm{I}}(\omega) \sim 1$ and $\beta_\ell^{\mathrm{I}}(\omega) \sim A^+_{\omega\ell}$.

Determining the mode functions

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- We have $\alpha_\ell^{\mathrm{II,III}}(\omega) = \frac{1}{(2\ell+1)r_1^{\ell+1}}\left[\ell\left(e^{i\omega r_1} + A^+_{\omega\ell}e^{-i\omega r_1}\right) + i\omega r_1\left(e^{i\omega r_1} - A^+_{\omega\ell}e^{-i\omega r_1}\right)\right]$ and $\beta_\ell^{\mathrm{II,III}}(\omega) = \frac{r_1^\ell}{2\ell+1}\left[(\ell+1)\left(e^{i\omega r_1} + A^+_{\omega\ell}e^{-i\omega r_1}\right) - i\omega r_1\left(e^{i\omega r_1} - A^+_{\omega\ell}e^{-i\omega r_1}\right)\right]$,
- For regularity, we propagate the solutions and retain $R^+_{\omega\ell} \to \beta_\ell^{\mathrm{II,III}}(\omega) r n_\ell(\omega r) \sim \beta_\ell^{\mathrm{II,III}}(\omega)\left(\frac{-(2\ell-1)!!}{(\omega r)^\ell}\omega\right)$, which can further be substituted.

Effective Entangling Photons

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- We have $R^{+}_{\omega\ell} \approx \frac{-i2^{\ell+2}(\ell-1)!(\ell+1)!}{(2\ell+1)(2\ell)!}\left(\frac{M}{r}\right)^{\ell}(M\omega)$ for $M \ll r \ll \omega^{-1}$. Although at low frequencies the white hole modes are essentially entirely reflected back into the black hole by the potential barrier $V(r)$, these modes fall off in r as the power law $\frac{1}{r^{\ell}}$ and, penetrate far beyond the peak of the potential barrier.
- Similarly, we obtain $R^{-}_{\omega\ell} \approx -\frac{i^{3\ell+1}2^{\ell+1}\ell!}{(2\ell+1)!}(\omega r)^{\ell+1}$ for $M \ll r \ll \omega^{-1}$, corresponding to low-frequency incoming waves from infinity essentially unaffected by the black hole and suppressed by the factor due to the angular momentum barrier.

Entangling Photons in Unruh Vaccum

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

For Unruh vacuum, $G_U^+(\omega) = \frac{1}{1-\exp(-2\pi\omega/\kappa)} \approx \frac{\kappa}{2\pi\omega}$ and $G_U^-(\omega) = \Theta(\omega)$.

$$\langle N \rangle_U^- = 4q^2 \int_{-\infty}^{\infty} \frac{d\omega}{\omega} \left| \hat{d}(\omega) \right|^2 \sum_{\ell=1}^{\infty} \frac{C_\ell \left(1 - \frac{2M}{r}\right)}{r^4} G_U^-(\omega) \left| R_{\omega\ell}^-(r) \right|^2$$

$$\sim \frac{16q^2}{9} \int_{1/T}^{1/\min(T_1,T_2)} \frac{d\omega}{\omega} \frac{d^2}{\omega^2} \frac{C_1 \left(1 - \frac{2M}{r}\right)}{r^4} \Theta(\omega)(\omega r)^4 \sim \mathcal{O}\left(\frac{q^2 d^2}{[\min(T_1, T_2)]^2} \right)$$

Entangling Photons in Unruh Vaccum

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

Similarly,

$$\langle N\rangle_U^+ = 4q^2 \int\limits_{-\infty}^{\infty} \frac{d\omega}{\omega}\left|\hat{d}(\omega)\right|^2 \sum_{\ell=1}^{\infty} \frac{C_\ell\left(1-\frac{2M}{r}\right)}{r^4} G_U^+(\omega)\left|R_{\omega\ell}^+(r)\right|^2$$

$$\sim \frac{128\kappa q^2 M^4}{9\pi} \int\limits_{1/T}^{1/\min(T_1,T_2)} \frac{d\omega}{\omega}\frac{d^2}{\omega^2}\frac{C_1\left(1-\frac{2M}{r}\right)}{r^4}\frac{1}{\omega}\frac{\omega^2}{r^2} \sim \mathcal{O}\left(\frac{q^2 d^2 M^3}{D^6}T\right)$$

For large T, $\langle N\rangle_U = \langle N\rangle_U^+ + \langle N\rangle_U^- \sim \mathcal{O}\left(\frac{q^2 d^2 M^3}{D^6}T\right)$.

Gravitational Analogue of Local Decoherence

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- For the stress-energy tensor of the first component of the particle being a point particle, $T_1^{\mu\nu} \approx \frac{m}{\sqrt{-g}}\delta^{(3)}\left(\underset{\sim}{x} - \underset{\sim}{X}_1(t)\right)u_1^\mu u_1^\nu \frac{d\tau_1}{dt}$. If conservation, then geodesic motion. External force implies a non-geodesic trajectory.
- Noting that the dipole contribution to the difference in stress energy of the components should be cancelled by the stress-energy effects of the lab.
- Analogous to the difference in the current densities, we have the leading order difference as $T_1^{\mu\nu} - T_2^{\mu\nu} \approx \frac{2m}{\sqrt{-g}}\frac{dt}{d\tau}t^{[\mu}s^{\gamma]}t^{[\nu}s^{\delta]}\nabla_\gamma\nabla_\delta\left[d^2(t)\delta^{(3)}\left(\underset{\sim}{x} - \underset{\sim}{X}(t)\right)\right]$.

Gravitational Analogue of Local Decoherence

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- Considering the quantum field observable corresponding to the electric part of the Weyl tensor as $E_{\mu\nu} = C_{\mu\gamma\nu\delta}t^{\gamma}t^{\delta}$.
- Thereby, for the decoherence, we note the variation in the equivalent potential $\mathbf{h}^{in}_{\mu\nu}(T^{\mu\nu}_1 - T^{\mu\nu}_2) \approx 2m \int dt\ d^2(t) s^{\mu} s^{\nu} \mathbf{E}^{in}_{\mu\nu}(t, \underset{\sim}{X}(t))$.
- Thus, the decoherence relates to obtaining the two-point function of the Weyl tensor. $\langle N \rangle = \langle \Psi_0 | \left(\mathbf{h}^{in}_{\mu\nu}(T^{\mu\nu}_1 - T^{\mu\nu}_2) \right)^2 | \Psi_0 \rangle \approx$ $4m^2 \int dt\ dt'\ d^2(t)\ d^2(t') \langle \Psi_0 | s^{\mu} s^{\gamma} \mathbf{E}^{in}_{\mu\gamma}(t, \underset{\sim}{X}(t)) s^{\nu} s^{\delta} \mathbf{E}^{in}_{\nu\delta}(t', \underset{\sim}{X}(t')) | \Psi_0 \rangle$.

Gravitational Analogue of Local Decoherence

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- From the analogous computations for electromagnetism, $\langle N\rangle \approx 4m^2 \int\limits_{-\infty}^{\infty} \frac{d\omega}{\omega}\left|\hat{d}^2(\omega)\right|^2 \sum_{\ell=2}^{\infty} \frac{C_\ell\left(1-\frac{2M}{r}\right)}{r^4}\left[G^+(\omega)\left|R^+_{\omega\ell}(r)\right|^2 + G^-(\omega)\left|R^-_{\omega\ell}(r)\right|^2\right].$
- Thereby, we have, for Unruh vacuum, $\langle N\rangle^-_U \sim \frac{m^2 d^4}{[\min(T_1, T_2)]^2}$ and $\langle N\rangle^+_U \sim \frac{M^5 m^2 d^4}{D^{10}} T$.

Interpreting the Decoherence

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

$$\langle N\rangle \approx 4q^2 \int dt\ dt'\ d(t)\ d(t') \left\langle s^\mu \mathbf{E}^{in}_\mu(t, \underset{\sim}{X}(t)) s^\nu \mathbf{E}^{in}_\nu(t', \underset{\sim}{X}(t')) \right\rangle_\Omega$$

$$\approx 4q^2 \left\langle \left(\int dt\ d(t) s^\mu \mathbf{E}^{in}_\mu \right)^2 \right\rangle_\Omega \sim 4q^2 d^2 T^2 \left[\Delta(s^\mu \mathbf{E}^{in}_\mu) \right],$$

where $\Delta(s^\mu \mathbf{E}^{in}_\mu) = \left\langle \left(\frac{1}{T} \int dt \frac{d(t)}{d} s^\mu \mathbf{E}^{in}_\mu \right)^2 \right\rangle_\Omega$ interpreted as the root mean square of the time average of the s^μ component of the electric field fluctuations in state $|\Omega\rangle$ on the worldline during the duration of the experiment.

Interpreting the Decoherence

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

Power spectrum, $S_U^r(\omega) = \int_{-\infty}^{\infty} dt \ \exp(i\omega(t-t')) \left\langle \mathbf{E}_r(t, \underset{\sim}{X}(t))\mathbf{E}_r(t, \underset{\sim}{X}(t')) \right\rangle_{\Omega_U}$.

Dominant contribution from $R^+_{\omega\ell}$ with $\ell = 1$ and $\omega \sim \frac{1}{T}$.

$$\langle S_U^r(\omega)\rangle \sim \int_{-\infty}^{\infty} dt \ \exp(i\omega(t-t')) \sum_{\ell=1}^{\infty} \frac{C_\ell \left(1-\frac{2M}{r}\right)}{r^4} G_U^+(\omega) \left| R^+_{\omega\ell}(r) \right|^2$$

$$\sim \frac{1}{r^4}\frac{1}{\omega} G_U^+(\omega) \left| R^+_{\omega\ell}(r) \right|^2 \sim \frac{\kappa}{r^4\omega^2}\left(\frac{M^2\omega}{r}\right)^2 \sim \frac{M^3}{r^6}.$$

Interpreting the Decoherence

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- Corresponds to the black hole in the Unruh vacuum acting as though it were an ordinary body with a randomly fluctuating electric dipole moment. $\mathcal{O}(\sqrt{h})$ effect independent of frequency suggesting contribution over all modes.
- Restoring the constants, the fluctuating electric dipole $\Delta|\underset{\sim}{\mathcal{P}_U}|(\omega) \sim \frac{\sqrt{\epsilon_0 \hbar} G^{3/2} M^{3/2}}{c^3}$.

  Similarly, for gravitational case, fluctuating quadrupole $\Delta|\underset{\sim}{\mathcal{Q}_U}|(\omega) \sim \frac{\sqrt{\hbar} G^2 M^{5/2}}{c^5}$.
- More generally, the power spectra of the higher electric multipole fluctuations and mass multipole fluctuations of the black hole go as $\Delta|\underset{\sim}{\mathcal{Q}_\ell}|(\omega) \propto M^{\ell+1/2}$.

  Dominant contribution from lowest electric parity multipole moment.

Decoherence in the Boulware Vacuum

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- Boulware vacuum $|\Omega\rangle_B$ ground state for the exterior region of Schwarzschild with respect to the timelike Killing field. We have $G_B^-(\omega) = G_U^-(\omega) = \Theta(\omega)$, hence $\langle N\rangle_B^- = \langle N\rangle_U^- = \mathcal{O}\left(\frac{q^2 d^2}{[\min(T_1,T_2)]^2}\right)$.
- Since $G_B^+(\omega) = \Theta(\omega)$, we retrieve $\langle N\rangle_B^+$ as

$$\sim \frac{256 q^2 M^4}{9} \int_{1/T}^{1/\min(T_1,T_2)} \frac{d\omega}{\omega} \frac{d^2}{\omega^2} \frac{C_1\left(1-\frac{2M}{r}\right)}{r^4} \frac{\omega^2}{r^2} \sim \mathcal{O}\left(\frac{q^2 d^2 M^3}{D^6} \ln\left(\frac{T}{\min(T_1,T_2)}\right)\right).$$

Note the $\ln T$ dependence arising, unlike the linear nature in Unruh case appearing as a result of $\ln V$ dependence with $V \sim \exp(\kappa T)$.

Decoherence in the Boulware Vacuum

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- Thereby, the expected number of entangling photons $\langle N\rangle_B = \langle N\rangle_B^+ + \langle N\rangle_B^- \sim \mathcal{O}\left(\frac{q^2 d^2 M^3}{D^6}\ln\left(\frac{T}{\min(T_1,T_2)}\right)\right)$.
- For $M\omega \ll 1$, we have $\Delta|\underset{\sim}{\mathcal{P}_B}|(\omega) \sim M^2\omega^{\frac{1}{2}}$ and $\Delta|\underset{\sim}{\mathcal{Q}_B}|(\omega) \sim M^3\omega^{\frac{1}{2}}$, suggesting ω dependence and $\Delta|\underset{\sim}{\mathscr{D}_B}|(\omega) \sim M^{\ell+1}\omega^{\frac{1}{2}}$ which are much smaller than the corresponding fluctuations in the Unruh vacuum.

Decoherence in the Hartle-Hawking Vacuum

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- Hartle-Hawking vacuum $|\Omega\rangle_H$ is a thermal state with respect to all modes at temperature $\mathscr{T} = \frac{\kappa}{2\pi}$. We have $G^+_{HH}(\omega) = G^+_U(\omega) = \frac{1}{1-\exp(\frac{2\pi\omega}{\kappa})}$, hence $\langle N\rangle^+_{HH.} = \langle N\rangle^+_U = \mathcal{O}\left(\frac{q^2 d^2 M^3}{D^6} T\right)$.
- We now have $G^-(\omega) = \frac{1}{1-\exp(\frac{2\pi\omega}{\kappa})} \sim \frac{\kappa}{2\pi\omega}$, thereby we get

$$\langle N\rangle^-_{HH} \sim \frac{8\kappa q^2}{9\pi} \int_{1/T}^{1/\min(T_1,T_2)} \frac{d\omega}{\omega} \frac{d^2}{\omega^2} \frac{C_1\left(1-\frac{2M}{r}\right)}{r^4} \frac{1}{\omega}(\omega r)^4 \sim \mathcal{O}\left(\frac{q^2 d^2 \kappa}{[\min(T_1,T_2)]}\right).$$

Decoherence in the Hartle-Hawking Vacuum

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- Thereby, Thereby, the expected number of entangling photons $\langle N \rangle_{HH} = \langle N \rangle^{+}_{HH} + \langle N \rangle^{-}_{HH} \sim \mathcal{O}\left(\frac{q^2 d^2 M^3}{D^6} T\right)$. We could always minimize the thermal contribution of incoming modes. Although the radiation incoming from infinity is thermal, it does not have the necessary population of soft modes.
- We have neglected the collisional decoherence of individual soft photons resulting in decoherence that grows with time due to the differential scattering. The decoherence rate due to emission of soft radiation falls off rapidly with distance from the black hole, whereas the collisional decoherence rate falls off more slowly in the Unruh vacuum and does not fall off at all in the Hartle-Hawking vacuum.

Decoherence in the Minkowski spacetime

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- In Minkowski spacetime, $R^{+}_{\omega\ell} = 0$, and we have the incoming modes from infinity $R^{-}_{\omega\ell} = -2i^{3\ell+1}\omega r j_\ell(\omega r)$. The Minkowski vacuum $|\Omega\rangle_M$ corresponds to $G^{-}_{M}(\omega) = G^{-}_{U}(\omega) = \Theta(\omega)$, hence the estimate $\langle N\rangle_M = \langle N\rangle^{-}_{M} = \langle N\rangle^{-}_{U} \sim \mathcal{O}\left(\frac{q^2 d^2}{[\min(T_1,T_2)]^2}\right)$, which can be nullified.
- If we thermally populate the $R^{-}_{\omega\ell}$ modes at temperature $\mathscr{T}$, the decoherence will be given by the same estimate of the Hartle-Hawking vacuum corresponding to $\frac{\kappa}{2\pi}$ as $\langle N\rangle^{\mathscr{T}}_{M} = \langle N\rangle^{-}_{HH} \xrightarrow{\mathscr{T}=\frac{\kappa}{2\pi}} \mathcal{O}\left(\frac{q^2 d^2 \mathscr{T}}{\min(T_1,T_2)}\right)$.

Scalar Field Decoherence in the Minkowski spacetime

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- In Minkowski spacetime, a memory effect and associated infrared divergences occur at null infinity for a massless field as a result of a permanent change in the field at order $\frac{1}{r}$.
- Charge conservation implies that such changes must be invoked by Lorentz boosting of Coulomb fields, which occurs pertinently in scattering due to differential momentum.
- For the local nature of the experiment, we expect changes in particle momentum to not last for a long enough time T to produce significant decoherence.

Scalar Field Decoherence in the Minkowski spacetime

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- For a scalar field, scalar charge need not be conserved, and a change in $\mathcal{O}(\frac{1}{r})$ can be achieved by simply changing the monopole moment of the source. Consequently, a source with a permanent change of scalar charge will radiate an infinite number of soft massless scalar particles in $\ell = 0$ modes.
- Suppose we have a massless scalar field ϕ in the background, and we perform the experiment with a scalar charge. Suppose, further, that the protocol includes changing the charge of one of the components during separation and then restoring the charge during the recombination.

Scalar Field Decoherence in the Minkowski spacetime

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- We have, analogously $\langle N \rangle = \langle \Omega | [\phi^{in}(j_1 - j_2)^2 | \Omega \rangle$. We further have the analogous expansion for the two-point correlation function as

$$\langle \Omega | \mathbf{E}_r^{\phi}(x) \mathbf{E}_r^{\phi}(x') | \Omega \rangle = \sum_{\ell=0}^{\infty} \frac{1}{16\pi^2} (2\ell+1) \frac{P_\ell(\hat{r}, \hat{r}')}{rr'} \int_{-\infty}^{\infty} \frac{d\omega}{\omega} \exp(-i\omega(t-t')) \times \left[G^+(\omega) R^+_{\omega\ell}(r) \overline{R^+_{\omega\ell}(r')} + G^-(\omega) R^-_{\omega\ell}(r) \overline{R^-_{\omega\ell}(r')} \right].$$

- Further, $\langle N \rangle_M^{\phi} \sim 4 \int_{-\infty}^{\infty} \frac{d\omega}{\omega} \left| \hat{q}_\phi \right|^2 \sum_{\ell=0}^{\infty} \frac{C_\ell}{r^2} G^-(\omega) \left| R^-_{\omega\ell}(r) \right|^2.$

Scalar Field Decoherence in the Minkowski spacetime

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- For the case where the scalar field initially is in the Minkowski vacuum state,

$$\langle N\rangle^{\phi}_{M} \sim \frac{8}{9}\int_{1/T}^{1/\min(T_1,T_2)} \frac{d\omega}{\omega}\frac{(\Delta q_\phi)^2}{\omega^2}\frac{C_0}{r^2}\Theta(\omega)(\omega r)^2 \sim \mathcal{O}\left((\Delta q_\phi)^2 \ln\left(\frac{T}{[\min(T_1,T_2)]}\right)\right).$$

- If Minkowski spacetime is initially filled with a thermal bath of scalar particles at temperature $\mathscr{T}$, we obtain,

$$\langle N\rangle^{\phi}_{M} \sim \frac{8}{9}\int_{1/T}^{1/\min(T_1,T_2)} \frac{d\omega}{\omega}\frac{(\Delta q_\phi)^2}{\omega^2}\frac{C_0}{r^2}\frac{\mathscr{T}}{\omega}(\omega r)^2 \sim \mathcal{O}\left((\Delta q_\phi)^2 T\mathscr{T}\right).$$

Decoherence in the Spacetime of a Static Star

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- Absence of the white hole modes in the case of a star causes differential behaviour of the field around the spacetime of the star. Additional degrees of freedom are associated with the presence of a horizon. The correlation function of the Minkowski spacetime is modified by the presence of a star, but the corrections are small for low frequencies with respect to the radius of the star: $\omega\mathcal{R} \ll 1$.
- Decoherence in the spacetime of a star with the electromagnetic field initially in its ground state is the same as the decoherence in Schwarzschild due to the incoming modes from infinity in the Boulware or Unruh vacua, which, in turn, is the same as the decoherence in Minkowski spacetime in the Minkowski vacuum.

Decoherence due to a Star with Internal Degrees of Freedom

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025]

- Internal degrees of freedom can couple to the components of the particle via ordinary interactions, and result in the decoherence if there is suitable dissipation in the material body system.
- Close to the horizon, the dominant contribution to decoherence comes from the $\ell = 1$ white hole modes at very low frequencies, which correspond to radiation and represent the equivalent additional degrees of freedom of the electromagnetic field. These resemble the exterior dipole field of an ordinary body, with a fluctuating electric dipole moment.

Decoherence due to a Star with Internal Degrees of Freedom

[Danielson, Daine L., Gautam Satishchandran, and Robert M. Wald., 2025, Biggs, Anna, and Juan Maldacena., 2024]

- Decoherence due to black holes using an effective theory is the same qualitative effect is present for ordinary matter at finite temperature. In this framework, the decoherence is seen to arise from thermal fluctuations of the multipole moments of the black hole/matter system.
- The Dipole moment of the black object has thermal fluctuations that are coherent on a timescale set by the quasinormal modes, which leads to a differential phase accumulation leading to decoherence,
- For the electromagnetic effect, the decoherence can be of equal magnitude for black holes and ordinary objects. For the gravitational effect, ordinary matter produces a much weaker effect than black holes of the same size.

Kerr and Reissner–Nordström metrics

[Gralla, Samuel E., and Hongji Wei., 2024]

- In Kerr metric, the leading order large T behaviour of the number of entangling photons is estimated on a bifurcate Killing horizon as $\langle N \rangle_{\mathcal{H}} \approx C \begin{cases} \kappa T & \kappa \neq 0 \\ 2 \ln T & \kappa \to 0 \end{cases}$, where $\kappa \to 0$ corresponds to $\kappa T \ll 1$.
- For Reissner–Nordström metric, we have the number of entangling photons as $\langle N \rangle_{\mathcal{H}^+} \approx C \kappa_{\mathcal{H}^+} \Delta T$, thus the entangling photon number will be sufficiently suppressed for extremal case, and the coherence of the spatially separated quantum superposition will be maintained.

Foundations

[Gralla, Samuel E., and Hongji Wei., 2024]

- For a Klein Gordon field $\Box\phi = 0$, with a Klein Gordon inner product evaluated on a Cauchy surface Σ (assuming globally hyperbolic), with $(\phi_1, \phi_2) = i\int_\Sigma (\phi_1^* \nabla_\mu \phi_2 - \nabla_\mu \phi_1^* \phi_2) n^\mu \sqrt{h}\ d^3x$, such that we obtain a complete set of modes with positive and negative frequencies.
- For a sourced theory, we have the Heisenberg picture $\Box\hat{\phi} = -4\pi\rho$, with quantization $\hat{\phi} = \sum_i (\hat{a}_i \phi_i + \hat{a}_i^\dagger \phi_i^*) + \phi^{\text{classical}}$, which can be seen as a quantum fluctuation about a classical inhomogeneous background.

Foundations

[Gralla, Samuel E., and Hongji Wei., 2024]

- At early time, we have the retarded classical solution with no incoming radiation, and at late times, we choose the advanced classical solution with no outgoing radiation. The difference is a source-free solution and can be expanded in modes.
- The difference $\hat{\phi}^{ret} - \hat{\phi}^{adv} = \sum_i (\alpha_i \phi_i + \alpha_i^* \phi_i^*)$ such that we define $\hat{\phi} = \sum_i (\hat{a}_i \phi_i + \hat{a}_i^\dagger \phi_i^*) + \hat{\phi}^{ret}$ such that we specify the early time Couloumb state $\hat{a}_i |\text{in}\rangle = 0$, and a similar expression $\hat{\phi} = \sum_i (\hat{b}_i \phi_i + \hat{b}_i^\dagger \phi_i^*) + \hat{\phi}^{adv}$ such that we specify the late times Couloumb state $\hat{b}_i |\text{out}\rangle = 0$.

Entanglement

[Gralla, Samuel E., and Hongji Wei., 2024]

- The *in* state has *out* particles given by $\langle \text{in}|\hat{b}_i \hat{b}_i^\dagger|\text{in}\rangle = |\alpha_i|^2$ by the evolution in the Schrodinger picture. Thereby, we realise the evolution $\hat{a}_i = \hat{b}_i + \alpha_i$, since both inhomogeneous solutions must differ by a radiation field.
- The total number of particles by the source is thereby, extracted by the positive frequency extraction map $K(\phi^{ret} - \phi^{adv}) := \sum_i \alpha_i \phi_i$, such that we have the total decoherence particles $\langle N \rangle = \sum_i |\alpha_i|^2 = \left(K(\phi^{ret} - \phi^{adv}), K(\phi^{ret} - \phi^{adv}) \right)$.

Setting Up

[Gralla, Samuel E., and Hongji Wei., 2024]

- In the Schrodinger picture, we assume semi-classical densities to treat each component of the field evolution as a classically sourced quantum field.
- At early times, we have $j_L = j_R$, and further at late times. Thereby, we have the Coulomb fields agreeing, $\phi_R^{ret} - \phi_L^{ret} = 0$ and $\phi_L^{adv} - \phi_R^{adv} = 0$.
- We let the quantum field undergo unitary evolution, from the initial vacuum state $|\psi_0\rangle = |\text{in}\rangle$. Since the retarded solution agrees at early times, the vacuum is unambiguously defined such that $\hat{a}_i|\psi_0\rangle = 0$.

Setting Up

[Gralla, Samuel E., and Hongji Wei., 2024]

- Initial State $|\xi\rangle = \frac{1}{\sqrt{2}} \underbrace{(|\Psi_{0L}\rangle + |\Psi_{0R}\rangle)}_{\text{Initial Matter State}} \otimes \underbrace{|\psi_0\rangle}_{\text{Unruh State}}$, with states that go left and right when the superposition is created, with $\langle \Psi_L | \Psi_R \rangle = 0$. Semi-classical analysis of the charge states with non-fluctuating density.
- Evolution $|\xi\rangle = \frac{1}{\sqrt{2}}(|\Psi_L\rangle \otimes |\psi_L\rangle + |\Psi_R\rangle \otimes |\psi_R\rangle)$, due to differential evolution causing entanglement. We aim to find the overlap $\langle \psi_L | \psi_R \rangle$.

Decoherence Analysis

[Gralla, Samuel E., and Hongji Wei., 2024]

- At late times, we note that $\lim_{t\to\infty} |\langle\psi_L|\psi_R\rangle| = |\langle\psi_0|\hat{\mathcal{D}}|\psi_0\rangle|$ where the difference in evolution is characterized by $\hat{\mathcal{D}} = \lim_{t\to\infty} \hat{U}_L^\dagger(t,t_0)\hat{U}_R(t,t_0)$ where $U(t,t_0)$ describes the evolution of the respective component.
- The field operators only differ in the classical component, hence $\hat{\phi}_R - \hat{\phi}_L = \phi_R^{ret} - \phi_L^{ret}$ at early times. Since the Coulomb field agrees at late times, we have $\Delta\phi := \phi_R^{ret} - \phi_L^{ret} = (\phi_R^{ret} - \phi_R^{adv}) - (\phi_L^{ret} - \phi_L^{adv})$ as the source free entangling radiation.

Decoherence Analysis

[Gralla, Samuel E., and Hongji Wei., 2024]

- Thereby, the displacement operator shifts the fields as $\hat{\mathcal{D}}^\dagger \hat{\phi}_L \hat{\mathcal{D}} = \hat{\phi}_L + \phi_R^{ret} - \phi_L^{ret} = \hat{\phi}_L + \Delta\phi$.
- Mode expanding, we have $\hat{\mathcal{D}} \simeq \exp\left[\sum_i \hat{a}_i^\dagger(\alpha_i^R - \alpha_i^L) - \hat{a}_i(\alpha_i^R - \alpha_i^L)^*\right]$ through the mode expansion coefficients of $\Delta\phi$.

Decoherence Analysis

[Gralla, Samuel E., and Hongji Wei., 2024]

- Coherence $\lim_{t\to\infty} |\langle\psi_L|\psi_R\rangle| = |\langle\psi_0|\hat{\mathcal{D}}|\psi_0\rangle| = \exp\left(-\frac{1}{2}\sum_i |\alpha_i^R - \alpha_i^L|^2\right)$.
- Thereby, the number of entangling photons produced by the difference in the sources is $\langle N\rangle = \sum_i |\alpha_i^R - \alpha_i^L|^2 = (K\Delta\phi, K\Delta\phi)$, thus the exponential decrease of coherence, $\lim_{t\to\infty} |\langle\psi_L|\psi_R\rangle| = \exp\left(-\frac{1}{2}\langle N\rangle\right)$.

Killing Time

[Gralla, Samuel E., and Hongji Wei., 2024]

- For the scope of the experiment, the affine time U relates to the Killing time parameterising the symmetry orbit u as $U = -\exp(-\kappa u)$.
- Now, noting the difference in the fields as $\Delta\phi = (\phi_R^{ret} - \phi_R^{adv}) - (\phi_L^{ret} - \phi_L^{adv})$ being the entangling field. We have the cases for late times where the advanced fields agree, resulting in $\Delta\phi = \phi_R^{ret} - \phi_L^{ret}$, and at eary timescales where the retarted fields agree, we have $\Delta\phi = -(\phi_R^{adv} - \phi_L^{adv})$.
- The Klein Gordon inner product for the number of entangling photons can be evaluated on any Cauchy surface in the globally hyperbolic region; we shall evaluate it at the past horizon.

Number of Entangling Photons

[Gralla, Samuel E., and Hongji Wei., 2024]

- For the positive frequency Unruh modes on the past horizon, we have the sum of the horizon and the null infinity contributions $\langle N \rangle = \langle N \rangle\big|_{\mathcal{H}^-} + \langle N \rangle\big|_{\mathscr{I}^-}$.
- For the past horizon, we parameterise with the Killing time u, and note the inner product $(K\Delta\phi, K\Delta\phi)\big|_{\mathcal{H}^-} = \frac{1}{\pi}\int d\mathcal{S} \int_0^\infty d\omega \; |\Delta\tilde{\phi}|^2 \omega \coth\left(\frac{\pi\omega}{\kappa}\right)$ where $\Delta\tilde{\phi}$ is the Fourier transform of $\Delta\phi$ with respect to the Killing time as $\Delta\tilde{\phi} = \int du \; \Delta\phi\big|_{\mathcal{H}^-} \exp(-i\omega u)$ on the past horizon.

Evaluation of the Integral

[Gralla, Samuel E., and Hongji Wei., 2024]

- For the wide stationery period of time T, we pick an intermediate frequency ω_c such that $\frac{1}{T} \ll \omega_c \ll \frac{1}{\mathcal{T}}$ where $\mathcal{T} = \max(T_1, T_2) \ll T$.
- In this regime, the Fourier transform reduces to a rectangular transform with $\Delta\tilde{\phi} \simeq \overline{\Delta\phi}\frac{2}{\omega}\sin\left(\frac{\omega T}{2}\right)$, such that the integral upto ω_c is evaluated.

Number of Entangling Photons

[Gralla, Samuel E., and Hongji Wei., 2024]

- We have $\frac{1}{\pi}\int d\mathcal{S}\int_0^{\omega_c} d\omega\ |\Delta\tilde{\phi}|^2\omega\coth\left(\frac{\pi\omega}{\kappa}\right) \approx \begin{cases} C\kappa T & \kappa\neq 0 \\ 2C\ln T & \kappa = 0\end{cases}$ as $T\to\infty$, where C is the constant of proportionality regarding the amplitude of flux.
- The number of entangling photons is thereby, $\langle N\rangle\big|_{\mathcal{H}^-} \approx C\begin{cases} \kappa T & \kappa\neq 0 \\ 2\ln T & \kappa = 0\end{cases}$ where the decohering flux is defined as $C = \int d\mathcal{S}\ |\overline{\Delta\phi}|^2$ for the Klein Gordon case, and $C = \frac{1}{4\pi^2}\int d\mathcal{S}\ \left(|\nabla_\Omega\nabla^{-2}E_r|^2 + |\nabla_\Omega\nabla^{-2}B_r|^2\right)$ for the electromagnetic case, with gauge fixing.

Number of Entangling Photons

[Gralla, Samuel E., and Hongji Wei., 2024]

- Thereby, the coherence $\left|\langle\psi_L|\psi_R\rangle\right| = \exp\left(-\frac{1}{2}\langle N\rangle\right)$ results in $\left|\langle\psi_L|\psi_R\rangle\right| = \begin{cases} \exp\left(-\frac{C}{2}\kappa T\right) & \kappa \neq 0 \\ \left(\frac{T}{T_0}\right)^{-C} & \kappa = 0 \end{cases}$, thereby a power law decay in the extremal case, and an exponential falloff in the non-extremal limit.
- The dependency of coherence on the decohering flux C will lead to an understanding of the falloff on the horizon.

Decohering Flux

[Gralla, Samuel E., and Hongji Wei., 2024]

- The decohering flux for Klein Gordon in Kerr through superposition of different monopole moments.
- For fixed electromagnetic charge in radial superposition, at a fixed proper distance d.

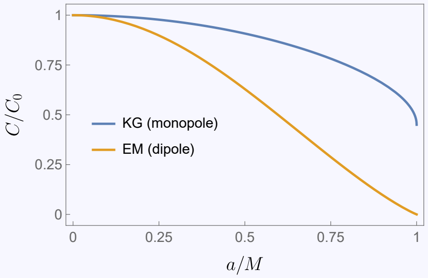

Figure: Decohering Flux

Gravitational Meissner Effect

[Gralla, Samuel E., and Hongji Wei., 2024]

- Interesting falloff of the electromagnetic decohering flux to null for the extremal case.
- Can be inferred from the screening of electric charges in the extremal Kerr black hole. Hence, no *which path* information penetrates the black hole.

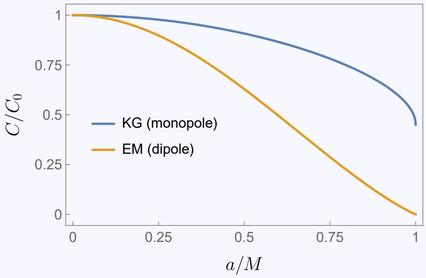

Figure: Decohering Flux

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Thank You!

Questions? Comments?